Travel time derivative, dynamic road toll on QOS—
market analysis and pricing issues

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August 1, 2009

Abstract

In this paper, travel time derivative is introduced as a new form of dynamic road
toll to change user behavior, a new funding tool for the traffic projects, a new hedging
tool against transportation related risk and a promising investment for investors. The
potential market is analyzed and the benefits for hedgers, speculators and providers are
enormous. Possible products are designed, possible prediction and pricing methods are
introduced.

Keyword: Travel time derivative, Travel time index, Travel time Options, High
Congestion Days, Low Congestion days. Traffic Default Obligations, No arbitrage
pricing, Non-tradable asset, Quality of service (QOS), Traveler behavior, Alternative Risk
Transfer (ART)

1 Introduction

Financial derivatives is any trading instrument whose prices are derived from the value
of something else (known as the underlying). The underlying can be tradable items
such as stocks and commodities, while also be non tradable items, such as weather (for
weather derivatives) or catastrophe (for catastrophe bonds). In this paper, we consider
the similarity between stochastic travel time and weather changes, and initiate the new financial derivative based on stochastic travel time—travel time derivative.

We believe, for transportation system, travel time derivative can act as a new funding source for various projects and an effective way to adjust network flow for important networks; for financial systems, the hedge and speculation with the new independent investment will make travel time derivative charming investment in future, especially the risk of traditional asset classes showed great risk in the recent financial crisis. It is time to look for risk diversification within other systems, and traffic systems will be a first choice.

The crucial questions for travel time derivative are as following:

1. What are the critical factors to enable a derivative market to come into being? Is it necessary to introduce a more complex derivative market based on stochastic travel time as that for weather (temperature and rain)?
2. How is the market formed and who are the potential participants?
3. What are the main products like?
4. What are the possible models to yield a reasonable price for a product? What kind of prediction models of travel time is suitable for pricing the travel time derivatives?

We try to raise these questions and initiate discussion in this paper. The paper is organized as follows:

Section 2 Background about derivative market, weather derivative and dynamic road toll and financial economy are presented. The needs from both financial sector and transportation sector are calling for the introduction of travel time derivatives.

Section 3 Conceptual market analysis of travel time derivative is given: Possible scenarios of trading are considered and potential market participants are listed.

Section 4 Derivative product design is discussed: The standard measurement of travel time is defined first which yields the accurate value of underlying travel time changes. Then travel time indices is introduced as the basic travel time based product. Classical derivatives are then given: simple options derived on the instantaneous value the experienced travel time; Options on the congestion days; Collateralized Travel Time Obligation (CTTO) based on correlated traffic time (such as the travel time in the urban network in Manhattan) can be defined.

Section 5 Prediction and Pricing for a basic product: We need a prediction model for travel time and a pricing model to get a final price. Different models and pricing
method are designed (PDE obtained from no arbitrage principle, indifferent pricing and hedge with correlated asset).

2 Initiation and necessity analysis

2.1 Derivative and weather derivative

Derivatives are financial instruments, whose prices are derived from the value of something else (known as the underlying). Any stochastic changing element which will generate changes of cash flow and related to economic life can serve as underlying asset here. Therefore, the underlying on which a derivative is based can be the price of an asset (e.g., commodities, equities (stock), residential mortgages, commercial real estate, loans, bonds), the value of an index (e.g., interest rates, exchange rates, stock market indices, consumer price index (CPI) see inflation derivatives), or other items.

The main types of derivatives are forwards, futures, options, and swaps, Hull[13]. Derivatives can be used to mitigate the risk of economic loss arising from changes in the value of the underlying. This activity is known as hedging. Alternatively, derivatives can be used by investors to take a risk and make a profit if the value of the underlying moves the way they expect (e.g. moves in a given direction, stays in or out of a specified range, reaches a certain level). This activity is known as speculation.

The underlying of derivatives can be classified as tradable and non-tradable, different derivatives are corresponding to different pricing method: the price of stock and other existing trading instrument are tradable, while for non-tradable underlying, weather has been a hot topic for the recent decade.

Weather derivative has been a hot topic in finance industry. As a non-tradable asset, a lot of derivatives based on weather has been designed. The market has been there since 1998. In 1999 the Chicago Mercantile Exchange introduced weather futures contracts, the payoffs of which are based on average temperatures at specified locations. Another example of derivatives with non-tradable underlings are catastrophe futures based on an insurance loss index regulated by an independent agency or simply derivatives based on equity indices such as S&P or DAX.

According to Stewart[22], In a wide variety of industries, from property management to natural gas retailing, firms face the possibility of significant earnings declines or advances because of unpredictable weather patterns. Weather derivatives offer an innovative hedging instrument to firms. Since 1997, weather derivatives have been written on several underlying variables including temperature, precipitation, snowfall, and windspeed in a myriad of forms including puts, calls, caps, floors, collars, and swaps.
Weather derivatives promise to have a tremendous impact on the economy because they are the first derivative instruments that hedge volume. Therefore, profits are directly affected. In light of the enormous potential, the Chicago Mercantile Exchange launched the first exchange traded weather derivative on September 22, 1999.

In Banks[1], various participants and roles in the weather derivative markets are analyzed. The business mode in the weather derivative market is formed in need of hedging, industries which are subject to weather risk all come and take roles in buy/sell side. Speculators then come in as an important source of liquidity. The trading and capital activities enable a beneficial mechanism for all.

Alternative Risk Transformation is the process of creating customized solutions that shift financial risks to the insurance and reinsurance markets, non financial risks to the derivative and capital markets and integrated programs to both, as given in [1].

The main pricing method for derivatives include several pricing models: Actuarial pricing, No arbitrage pricing(Market pricing), Indifferent pricing and Risk neutral/replication pricing etc.

1. Actuarial pricing: It accounts only for the statistics of the contract payout and are used to when pricing contracts on risks that are not tradable in a two-way marketplace. That is the underlying is not tradable, such as weather and travel time. The price of a financial instrument is the discounted sum of its expected future return and the additional price for risk determined by the contract properites which are based on the current price and position. [1]

   This method needs to understand the statistics of the contract payout and their relation to the current holding position. The modeling of the expected return and the risk is important and subject to choice of model.

2. Risk neutral/replication (Black-Scholes methodology) It assumes the underlying is tradable and we can use dynamic hedging to replicate the payoff of the derivative under the underlying assets, Steele[21]. However, this does not work for non-tradable assets such as weather and travel time.

3. Non arbitrage pricing: No arbitrage principle says "if two investment yields the same payoff in all scenarios, they must yield the same market price, other wise you will get arbitrage by buying the cheap and sell it high". It is more general and pricing by replication is based on this principle. Furthermore, even for non tradable assets we can use no arbitrage principle to derive the price for its derivatives. Consider the some research on no arbitrage pricing of interest rate derivatives, Sundaresan[23], we propose this way can be used for travel time derivatives Delbaen[8] Hull[11].
4. Indifferent pricing The indifferent pricing says "The utility indifference buy (or bid) price $p$ is the price at which the investor is indifferent (in the sense that his expected utility under optimal trading is unchanged) between paying nothing and not having the derivative and paying $p$ now to receive the payoff of the derivative at time $T$." This is also effective way to do pricing for weather derivatives, Carmona [4], Xu [29].

What calls our attention is the similar stochastic nature between temperature changes at a certain location and the travel time on the same place. And also the usual strong correlation between weather and travel time. We therefore propose the possibility of introducing travel time derivative. The benefits will be analyzed below.

2.2 An effective tool to change user behavior

Problem of congestion pricing has been studied in the literature from different modeling perspectives and under various assumptions. A classification can be made depending on different criteria: pricing theory (marginal cost pricing vs. second best pricing), objectives to be reached (minimizing total travel time or maximizing net social welfare or maximizing revenues), type of analysis (static or dynamic pricing), pricing strategy (link-based, path-based or zone-based pricing) and user classes (users can be classified based on travel cost perceptions, information access, value of time or vehicle category).

In Region [20], there is great interest in developing pricing models for congestion relief. However, most of the work in the literature uses static transportation models for analysis. The benefits of accounting for traffic dynamics under congestion pricing are unclear. Their research performs a systematic comparison of static traffic assignment with the VISTA model, a simulation-based dynamic traffic assignment approach, and with an approximation to DTA using an add-in for TransCAD software. Viti (2003) [27] gives a dynamic congestion-pricing model is formulated as a bi-level program, and the prices are allowed to affect the (sequentially modeled) route and departure time choice of travelers.

In traditional road toll, fixed to a road It is hard to evaluate to what extent this changes the traveler’s behavior. Time-varying toll is later promoted when different charge is given to different time. These ways do raise money for the road system but are not effective enough in changing traveler’s behavior. One step further, travel time derivative used in this context can be viewed as a dynamic toll associated to given quality of service. The market pricing and risk-return relation will change the traveler in a decentralized way and it is more realistic and efficient than optimizing the centralized objective as a planner.
An advantage of the travel time derivative is its high leverage, if a traveler buys travel time derivative with high leverage, that is, if the travel time on the link is as he expected he will get 200% profit, this return will change his behavior with high probability. However, if the leverage is too high, some travelers may take extreme behavior to achieve their own benefits. It will be an interesting behavior problem to study. In all, the leverage effect in derivatives is strong enough to change the behavior of the travelers.

2.3 A profitable tool for different participants and investors

To do analysis on possible market participants we need to identify some basic properties of the derivative which we are working on:

In Jewson[12], there are some attributes of weather derivatives, and travel time derivative has its own attributes as follows.

1. The contract period: the start date and an end date
2. A measurement segment/network: this is defined as the area which the measurement is taken
3. A travel time variable, measured at the measurement segment/network: mostly it is travel time, number of extreme travel time or number of days in which extreme travel time occur.
4. A standard measure method: travel time on the specific area can be measured more flexibly than weather. A standard measurement needs to be defined.
5. An index: it aggregates the travel time variable over the contract period in some way.
6. A pay-off function which converts the index into the cash flow that settles the derivative shortly after the end of the contract period
7. For some kind of contract, a premium paid from the buyer to the seller at the start of the contract.

It is supplemented by the necessary market elements:

1. A measurement agencies: responsible for measuring the travel time variable. Multi agencies which are in need to get an accurate measurement.
2. A settlement agent: responsible for producing the final payoff. It also defines how to deal with the eventualities and defaults.
3. A time period over which the settlement takes place.

There are two general types of products:

Type A travel time derivative: When the travel time is high, a leveraged reward is given. Type B travel time derivative: When the travel time is low, a leveraged reward is given. Their risk profiles are shown in Figure 1 and it shows also industries with different risk profiles will buy corresponding travel time derivatives to hedge their risk. [Insert Figure 1 here]

![Figure 1: Payoff and Risk Management with Travel time derivatives](image)

In detail, the roles and benefits for different participants can be analyzed according to their different needs to these two types. Detailed in Section 4.

### 2.4 Economy, Policy and Other initiatives

A more profound question is to estimate how much percentage of total economy is related to transportation service. According to the survey conducted by the U.S. Department of Commerce in 2004 estimates that approximately 30% exposed to some type and degree of weather risk, Finnegan (2005)[9]. For transportation the percentage is hard to estimate. There is no economy theory to answer what is the threshold for such percentage beyond which a derivative market can be initiated. Hence, the conclusion may differ from industry to industry. As transportation has been an important fundamental industry in economic life, we do hope travel time derivative market can come into being to show its influence.

In general, there are different determining factors that initiate different existing derivative markets. For the weather derivative, it is that a considerable percentage of
GDP is related to weather risk as suggested above and the primary needs for hedging is enormous. Emission trading comes into being as there is policy reinforcement on the emission of CO2, the forms have to buy licence for CO2 emission and can trade if the supply does not meet demand [Tietenberg (1985)][24] and [Jotzo][14]. Besides, the needs for new tools to diversify the risk and get leverage may also introduce new derivatives. The emerging of travel time derivative should be a mixture of various factors. We suggest a careful examination of all the factors and a debate on policy reinforcement, so that the market can finally run in a regulated and efficient framework.

3 Market analysis and Participants

In this section, we summertime some of the potential participants of travel time derivative markets. They are shown in the Figure and details are in the subsections below.

[Insert Figure 3 here]

3.1 Buyer of Type A: Participants who get benefit from good traffic condition and get hurt from bad traffic conditions.

1. Individual travelers They experience the quality of service of traffic system, and the bad performance of the system will cause delay and possible economic loss to them. More concretely, if the employees is late because of the traffic, they get a fine from the company. They can buy travel time derivatives which give them payment (type A) if the traffic is bad. If the traffic is bad, the company get lose in business, it should buy the type A derivative. Employers can buy such derivatives to act as traffic delay insurance for their employees or as compensation on bad transportation service. The scenario analysis for travelers in HCD call options (defined in later sections) are given in Figure ??, similar analysis can be conducted for other participants.

[Insert Figure ?? here]

2. Cargo transportation/Logistic/Deliver service If travel time is high, their level of service will go down and some urgent need for clients can not be met, there is a loss. If the traffic system gives bad performance the deliver service level
will tend to degrade and additional cost will increase. If Cargo transportation companies want to hedge such risk, they can buy Type A product.

3. **Tourism industry** Tour agencies will suffer economic loss if the traffic status is bad in a city, as the bad traffic condition may reduce the travelers’ image of the city and lead to reduction of traveler number in future. For example, if the overall service level of traffic is low in Mahatten, the experience of travelers will be bad, compensations can be made.

Tour agencies will be interested in the overall traffic status in the famous area in the country, an travel time derivative based on an average index will be ideal for them. Here the average index may be built on the performance on Manhattan, Chicago downtown, Los Angels downtown, etc.

4. **Event organizers** The transportation service during important event such as Olympics Games and Important International conference is of great interests. Travel time derivative during these events is valuable to raise fund for that event and also get compensations for the delay to participants.

A typical case is the Olympic Games. In Beijing, if we estimate the total traffic performance in the Chang-an Street in Beijing. If the average travel time during the Olympic Games on the street is greater than a given number K, a cash flow is paid as compensation for the buyer. The organizers can use this derivative to raise fund for organization of the event. Since different people have different expectation on such quality of service, it will form a market.

5. **City management** Overall traffic service level control is an important issue for a city. If the average travel time in an important district of city is high, the buyers get a compensation. It is used for the control of congestion and a advanced way of congestion pricing.

6. **Insurance companies**

Since accident number in an area is positively correlated to the experienced travel time in the same area, travel time derivative is a reasonable hedging tool for the insurance company itself. Furthermore, as a new insurance on experience travel time (quality of service of traffic system), they can issue new issuance product over travel time, to hedge out the risk brought by such sold product, issuance companies need to buy Type A travel time derivatives. It generally is type A derivatives.
3.2 Buyer of Type B: Participants who get benefit from bad traffic condition and get hurt from good traffic conditions.

1. **Gas company**
   Gas consumption is negatively correlated with average travel time. More gas is consumed if the traffic is slow. The gas companies will buy type B. The gas companies may need compensations if the traffic status is good.

2. **Car Maintenance** Since accident number get more business when the traffic network is bad, more car is used in a low speed condition and more maintenance is needed. They will buy type B.

3. **Automobile companies** When the traffic system is of low quality, the sales of automobiles will be affected. There is a potential loss for automobile companies. On the other hand, when the travel time is generally high, there tend to be more car accidents and the devaluation and impairment of sold cars will be faster, so more potential sales might be available, there is a potential profit. Therefore for automobile companies, both A/B type can be in need depending on the balance between the two factors above.

4. **Public transportation and Taxi companies**
   Buses and other public transportation runners will suffer from loss if the traffic status is bad. They tend to buy type A travel time derivative to hedge.
   Taxis run all day long in the road network. If the traffic system is bad, the average number of guest a taxi can serve in a given period will be reduced, then there is an economical loss. Therefore taxi companies can by type A derivatives to get compensations.

5. **Alternative transportation firms** There are many ways to travel, train transportation companies can buy the derivative on road transportation quality, to hedge their risk. If the quality of service of road travel time is better than before and more travelers choose to travel by car, the train transportation companies can get compensations for their losses. They generally need type B derivatives.

3.3 Other participants and general benefit consumer:

1. Banks Banks will be an important market maker, and as an independent new business area, travel time derivatives will be useful for banks to diversify portfolio risk.
2. Investors

We consider the alternative risk transfer relationship between transportation related industries and financial industry as shown in Figure 4. [Insert Figure 4 here]

From the financial system aspect, investors may invest in travel time derivatives for the following reasons:

(a) Invest in a new and independent resources to conduct investment or speculate.

The various products equities debt and currencies in various markets are becoming increasingly correlated. The issuance of large amounts of sovereign and corporate debt, coordination on international monetary policy, investment across markets and borders, implementation of quantitative arbitrage strategies driven by powerful computing resources, indexation of derivative products allowing large funds to quickly switch between asset classes and dissemination of critical financial news on a global real time basis have caused investors to seek investment strategies that generate returns. The travel time derivatives can be such a new area to diversify risk as it changes independently w.r.t traditional asset class. This is just a new resource of randomness where investors secure their fund after the great financial crisis in 2008. Type A/B can be both in need.

(b) **Hedging for weather risk**

Since the weather condition is correlated with the experience travel time, travel time derivative will be a good hedging tool for the investors who invest in the weather derivative market. Type A/B can be both in need.

(c) **Hedging for CO2 emission** Since the CO2 emission is highly correlated to traffic system performance, travel time derivative will be a good hedging tool for the CO2 emission market. The precondition is there is a tradable emission permission in the area, and the emission can be traded in the CO2 emission market. US just pass the policy to enable CO2 emission trading, so
the trading about this will be of interesting. If there is a derivative on the quality of air (it can be derived on CO2 percentage) travel time derivative will be a good hedge tool. Type A/B are both in need.

3. **Transportation project**

The transportation sector can initiate the market of travel time derivative, and public agencies can gain new funding through the trading of travel time derivatives. The invested money is then used to fund the undergoing transportation project. Besides, the demand to measure stochastic travel time in more dense and accurate way provide the demand to further develop transportation infrastructure. The demand together with the funding come from investment is of great help to the future development of the transportation systems.

4. **Manufacturers of transportation related products**

The sales is related to the status system. If the traffic system is yielding bad performance and there are too many cars, the sales of cars will be affected, and car manufacturing companies can use travel time derivative to hedge the risk.

For products which really are related to the real time status of the traffic system, travel time derivatives can be utilized to promote sales. For example, GPS companies can use the following strategies to promote the sales and coverage of GPS devices. Three modes can be used here:

(a) The company and user sign a contract, the user pays a risk premium, if the experienced travel time is high, the company pays back the user a certain percentage of the product price as a final reward, otherwise the sum of risk premium is just the product price which the user pays.

(b) The company lending a GPS devise to one user first, he uses it and pays premium until default occurs, the price of the device will be the sum of premium up to the default, after that he can switch to buy real travel time derivative.

(c) After selling the product, the company can buy the user a travel time derivative, if with the equipment, the user still experience high travel time than those without, a cash flow can be paid back by the contract which covers the expenses of the user.

The role of GPS companies includes also:

(a) Provide the fundamental measurement of travel time and gain income from both the travelers and the investors. Good advertisement as well.

Get ad-
ditional income from both travelers (a percentage of risk premium) and inves-
tors (fees for measurement).
(b) Conduct data validation and ensure sufficiency. Advanced data-gathering
mechanism and higher data quality should be achieved so that derivative
pricing can be backed by accurate data.

4 Product design

In this section we will design some basic forms of travel time derivatives. There payoff
are all different functions of the experienced travel time(Quality of Service of traffic
system) at a future time point or within a future time window. The direct association
of QOS to return will be a effective way to change traveler’s behavior and improve
traffic system.

When define the products, we need to define a standard measurement of travel time
at a certain time point

Definition 1. Standard measurement of travel time in a given time period The average
of all reported travel time from the specific travel time data providers on a specific path
which starts in specific time period on the specific day.

The average of all reported travel time from specific travel time data providers on
a specific path which starts within specific time period in the specific day. All the
time point in the following definitions are based on the standard measurement, and
the starting time point of the time window will be used as characterizing time.

4.1 Travel time index and equivalent return rate

The first design is to get an weighed average of the latest travel time in downtown
of main cities in States and formed a US realtime travel time index.It will be a nice
base for trading, good symbol for the transportation industry and good advertisement
for data providers(GPS companies). Other local index can be setup for local traffic
service.The definition is given below:

Definition 2. Travel time index

\[ T_{us} = \sum T_i \cdot \alpha_i \]

where \( T_i \) is the travel time in selected places in states.
An illustrating index can be formed by the weighted average of the realtime travel time from Broadway in New York (from 20th street to 60th street), Downtown Chicago, Downtown Los angels and Downtown Houston. This index can be viewed as a average traffic QOS index, which shows the national service level of urban traffic systems. The return rate of the index can be used as an average return rate when pricing travel time derivatives.

4.2 Design of derivative product on travel time

Type 1 Basic Option for a specific link at a given future time point

The simple derivative for the experienced travel time at a future time is defined as follows, and an example is given afterwards. Besides, the payoff graph are similar as the payoff in Figure 1.

**Definition 3.** Call option on a certain travel time Consider the link a, a specific time instant \( t \) in the future, the travel time experienced by a traveler entering at \( t \) (denoted as \( T \)) is higher than a given \( K \) then then there is a payment \( \alpha(T - K) \) to the option buyer; if lower, there is no payment. \( \alpha \) is the leverage coefficient.

**Definition 4.** Put option on a certain travel time Consider the link a, a specific time instant \( t \) in the future, if the travel time experienced by a traveler entering at \( t \) (denoted as \( T \)) is lower than a given \( K \) then then a payment \( \alpha(K - T) \) to the option buyer; if higher, there is no payment. \( \alpha \) is the leverage coefficient.

Consider Broadway NY from 20th street to 60th street, if the experienced travel time on it entering at 10:00 Jan-1 2011 is above 70 minutes and the threshold value is set as 60, then there is a leveraged cash back to the buyer $ 10 * (70 - 60), if the experienced travel time is lower than 60, the buyer get nothing.

Type 2 Congestion-days Option on the average performance of a specific path in a future time window. Congestion Day Options is the options based on the average performance of a path in a future time window, the definitions are given below, followed by an example. Besides, the payoff graph are similar as the payoff in Figure 1.

**Definition 5.** HCD and LCD Congestion-day. (congestion-days) Let \( T_i \) denote the mean travel time for day \( i \) and \( C \) as a specified travel time. We define the high congestion-days, \( HCD_i \), and the lower congestion-days, \( LCD_i \), generated on that day as \( HCD = \max(T_i - C, 0) \) and \( LCD = \max(C - T_i, 0) \) respectively.
Define $H_n = \sum_{i=1}^{n} HCD_i$; and $L_n = \sum_{i=1}^{n} LCD_i$

**Definition 6.** Call Options on high congestion days. Denote $K$ as the strike value:

The payoff of a HCD call is

$$X = \alpha \max(H_n - K, 0)$$

The payoff of a LCD Call is

$$X = \alpha \max(L_n - K, 0)$$

**Definition 7.** Call Options on high congestion days. Denote $K$ as the strike value:

The payoff of a HCD put is

$$X = \alpha \max(K - H_n, 0)$$

The payoff of a LCD put is

$$X = \alpha \max(K - L_n, 0)$$

Consider Broadway NY from the 20th street to the 60th Street, if the mean travel time on it is above 60 minutes we record a surplus $T - 60$ as a congestion day, otherwise we record 0 surplus. Then all these surplus values are added together for a whole year with 365 days. If the sum $S$ equals 2000 and so is larger than $K = 1500$, then there is a leveraged cash back $10 \times (S - K)$ where 10 is the leverage ratio, if not, the buyer get nothing. This is an example of HCD call option, examples for other options can also be given.

**Type 3 A Collateralized Travel Time Obligation to evaluate the performance of a traffic network in future time window.**

To associate the quality of service of a dense road network with certain payoff, there should be more complex mechanism. we consider its similarity to the Collateralized debt obligation(CDO) with a default pool of bonds.

**Definition 8. Collateralized Travel Time Obligation (CTTO)**

Consider a traffic network made of $N$ links, in which the link travel time are correlated. Consider a time window $[t_1, t_2]$, we call a default occurs on link $i$ if and only if the experienced travel time on the link is more than a pre-specified value $K_i$. Then the
number of defaulted links among the all $N$ link in a time period is counted and sliced into tranche and sold to different protection sellers.

Tranche 1: $0 \leq \frac{N}{m}$
Tranche 2: $\frac{N}{m} \leq \frac{2N}{m}$

$\ldots$

Tranche $m$: $\frac{N(m-1)}{m} \leq N$

The protection buyer pay constant premium $r$ to the buyer over time, if defaults occur within the range of a certain tranche, the corresponding protection sellers will pay $\alpha$ to all the buyers who buy the defaulted link. In this way, the protector buyer will get issuance on the quality of service of the traffic network. Usually, investors will act as protection seller while travelers can act as protection buyer. The quality of service of the road network is insured by such risk sharing mechanism.

Consider a CTTO for the manhattan road grid which is made of the 20th street to the 60th street, and the 2nd avenues and the 9th avenue. We can conduct the following slices:

- Tranche 1: 0–30 of the links in all will default.
- Tranche 2: 30–60 of the links in all will default.
- Tranche 3: 60–90 of the links in all will default.
- Tranche 4: 90 and above of the links in all will default.

Those who are optimistic about the QOS of the network will buy the Tranche 1:0–25 while those who think the network is generally congested will buy Tranche 4. To get a reasonable price and right investment choice, people have to consider the correlation between the links. Besides, the price of the tranches will also be based on historical experience of the network as well as people’s anticipation about the QOS of the traffic network.

The product based on the travel time indices can also be developed similarly. Index-based derivative will be less affected by one certain location and harder to be manipulated, as it is related to broader areas.

5 Pricing of Derivatives on travel time

There should be an appropriate prediction model and a corresponding pricing model for any forms of travel time derivatives. Simple options and options on congestion days can be studied with the knowledge in option-pricing theory. To price travel time default obligation for a traffic network. Consider the joint distribution of the arcs in the area and count the number of arcs in which defaults occur within a certain time
period in the area and selling protection against such defaults. Pricing of it is similar to a CDO. The underlying prediction model should be carefully picked, as the stochastic nature of travel time process is different from that of the asset return. Below we will show a model within normal framework and later the drawbacks will be indicated.

CDO pricing [15] [3]

5.0.1 Prediction


A model for pricing, should be able to approximate some basic properties of the travel time process as follows:

1. The travel time process is mean reverting, in other word, fluctuate around its mean value.

2. The probability change over time lag $s P(X_{t+s} = y | X_t = x)$ for fixed $t$ is also an important characteristic for travel time process. as shown in the travel time data from Los Angels, in Figure 5.

[Insert Figure 5 here]

The traditional models reviewed above are not enough for pricing purpose, while a stochastic process model can achieve both goals and the dependence and probability structure over time can be fully described by the model once the parameters are selected. Therefore, we consider some general Ito type processes models, and use to them to describe the travel time process:

$$dT_t = a_t(b_t - T_t)dt + \sigma_t dW_t$$

This model is a mean-reverting model for the link travel time $T_t$ around $b_t$ which means the travel time along a link is oscillating around its mean. $a_t$ is the reverting
rate, which shows how fast the process come back to its mean. \( \sigma \) is the volatility of the travel time process and \( W_t \) is Brownian motion, which has more complex structure than the usual i.i.d. white noise series. We then try to solve this model as follows:

Define \( x_t = T_t - b_t \)

\[
dx_t = -a_t x_t dt + \sigma_t dW_t
\]

Set

\[
y_t = e^{\int_0^t a_s ds} x_t
\]

Then

\[
dy_t = a_t e^{\int_0^t a_s ds} x_t dt + e^{\int_0^t a_s ds} dx_t
\]

\[
= a_t e^{\int_0^t a_s ds} x_t dt + e^{\int_0^t a_s ds} dx_t
\]

\[
= a_t e^{\int_0^t a_s ds} x_t dt - e^{\int_0^t a_s ds} a_t x_t dt + e^{\int_0^t a_s ds} \sigma_t dW_t
\]

\[
= e^{\int_0^t a_s ds} \sigma_t dW_t
\]

so

\[
y_t = y_0 + \int_0^t e^{\int_u^t a_s ds} \sigma_u dW_u
\]

with \( y_0 = e^0 x_0 = x_0 \)

So

\[
e^{\int_0^t a_s ds} x_t = x_0 + \int_0^t e^{\int_u^t a_s ds} \sigma_u dW_u
\]

\[
x_t = e^{-\int_0^t a_s ds} [x_0 + \int_0^t e^{\int_u^t a_s ds} \sigma_u dW_u]
\]

\[
T_t - b_t = e^{-\int_0^t a_s ds} [(T_0 - b_0) + \int_0^t e^{\int_u^t a_s ds} \sigma_u dW_u]
\]

\[
T_t = e^{-\int_0^t a_s ds} [T_0 - b_0] + b_t + e^{-\int_0^t a_s ds} \int_0^t e^{\int_u^t a_s ds} \sigma_u dW_u
\]

\[
T_t = T_0 e^{-\int_0^t a_s ds} + b_t - b_0 e^{-\int_0^t a_s ds} + e^{-\int_0^t a_s ds} \int_0^t e^{\int_u^t a_s ds} \sigma_u dW_u
\]
When the coefficients are constant, it reduces to the solution

\[ T_t = T_0 e^{-at} + b(1 - e^{-at}) + \int_0^t e^{-a(t-u)} \sigma dW_u \]

Q.E.D

Then we can calibrate the travel time process model by considering the conditional probability surface of the empirical conditional distribution.

For any time the distribution is Gaussian with mean and variance

\[ \mu_t = T_0 e^{-\int_0^t \sigma ds} + b_0 e^{-\int_0^t \sigma ds} \]

and

\[ \sigma^2_t = E(e^{-\int_0^t \sigma ds} \int_0^t e^{\int_0^u \sigma ds} \sigma dW_u)^2 \]

\[ = e^{-2 \int_0^t \sigma ds} \int_0^t e^{2 \int_0^u \sigma ds} \sigma^2 du \]

if the coefficients \( a_t \) and \( b_t \) are all time invariant, we get:

\[ \mu_t = T_0 e^{-at} + b(1 - e^{-at}) \]

\[ \sigma^2_t = E(\int_0^t e^{-a(t-u)} \sigma dW_u)^2 \]

\[ = \int_0^t e^{-2a(t-u)} \sigma^2 du \]

\[ = \frac{\sigma^2}{2a}(1 - e^{-2at}) \]

We equal the theoretical mean and variance to those of the empirical conditional distribution with the same time lag and then the parameters can be estimated. This model is actually try to use a sequence of normal distribution to approximate the probability surface above.

5.0.2 No arbitrage pricing model: hedging by portfolio of derivatives

Since travel time is a non-tradable asset, to price travel time derivative is similar to price interest rate derivatives and weather derivatives. Several ways can be used:

We propose three different ways and try to compare them:
1. Non arbitrage pricing by a portfolio of travel time derivatives. A PDE can be derived, similar to the interest rate derivatives.

2. Indifferent pricing: use a stochastic control scheme, and solve the HJM equations. It is flexible w.r.t different kinds of utility functions used in the context.

3. Direct partial hedge: solve a merton type problem with an asset which is correlated with the travel time changes.

4. For Collateralized Travel Time Obligation, The expectation based on the estimated joint distribution is a good pricing tool\cite{28} and the pricing is similar to the methods of pricing a Credit Default Obligation.

In this paper, we try to use no arbitrage pricing to price a simple travel time option as an illustration, other methods can also tried in future research.

Consider two travel time derivatives with different payoff functions: $F$ and $G$ We consider a portfolio $P = \alpha F + \beta G$ of $F$ and $G$ and $\alpha + \beta = 1$ suppose the underlying process of the travel time changes is $dX = A(t, X_t)dt + B(t, X_t)dW_t$. If we make the portfolio risk neutral then it should increases as the same rate as the interest rate The deductions is as follows

$$\alpha(\frac{\partial F}{\partial t} dt + \frac{\partial F}{\partial X} dX + \frac{1}{2} \frac{\partial F^2}{\partial X^2} d < X >) + \beta(\frac{\partial G}{\partial t} dt + \frac{\partial G}{\partial X} dX + \frac{1}{2} \frac{\partial G^2}{\partial X^2} d < X >)$$

$$\{\alpha(\frac{\partial F}{\partial t} + \frac{\partial F}{\partial X} A(t, X_t)+\frac{1}{2} \frac{\partial F^2}{\partial X^2} B^2(t, X_t)))+\beta(\frac{\partial G}{\partial t} + \frac{\partial G}{\partial X} A(t, X_t)+\frac{1}{2} \frac{\partial G^2}{\partial X^2} B^2(t, X_t))\} dt+\{\alpha \frac{\partial F}{\partial X} B(t, X_t)+\beta \frac{\partial G}{\partial X} B(t, X_t)\}$$

make it riskless we have:

$$\alpha \frac{\partial F}{\partial X} B(t, X_t) + \beta \frac{\partial G}{\partial X} B(t, X_t) = 0$$

And we have

$$\frac{\alpha}{\beta} = -\frac{\partial G}{\partial X}/\frac{\partial F}{\partial X}$$

Furthermore, the increasing rate should therefore be interest rate $r$:

$$\alpha(\frac{\partial F}{\partial t} dt + \frac{\partial F}{\partial X} A(t, X_t)+\frac{1}{2} \frac{\partial F^2}{\partial X^2} B^2(t, X_t)))+\beta(\frac{\partial G}{\partial t} dt + \frac{\partial G}{\partial X} A(t, X_t)+\frac{1}{2} \frac{\partial G^2}{\partial X^2} B^2(t, X_t)) = r$$
and we have
\[ \frac{\partial F}{\partial t} dt + \frac{\partial F}{\partial X} A(t, X_t) + \frac{1}{2} \frac{\partial^2 F}{\partial X^2} B^2(t, X_t) - r = \frac{\partial G}{\partial t} dt + \frac{\partial G}{\partial X} A(t, X_t) + \frac{1}{2} \frac{\partial^2 G}{\partial X^2} B^2(t, X_t) - r = \lambda \]
where we define \( \lambda \) as the market value of risk.

For individual travel time derivative with payoff \( F \), its price should satisfy the following PDE:
\[ \frac{\partial F}{\partial t} dt + \frac{\partial F}{\partial X} (A(t, X_t) - \lambda) + \frac{1}{2} \frac{\partial^2 F}{\partial X^2} \sigma^2_t = r \]
if we consider the model to be of the O-U process given in the previous section, we then can solve the PDE to get the price for the price of travel time derivatives.
\[ A(t, X_t) = a_t(b_t - X_t) \]
\[ B(t, X_t) = \sigma_t \]
The PDE is then
\[ \frac{\partial F}{\partial t} dt + \frac{\partial F}{\partial X} (a_t(b_t - X_t) - \lambda) + \frac{1}{2} \frac{\partial^2 F}{\partial X^2} \sigma^2_t = r \]
For call option with the payoff function is \( F = \max(0, X - K) \). With appropriate boundary conditions, this PDE can be solved and yield a proper price for the options under the model. Calibration of the parameters can be conducted by fitting the mean and variance for the conditional distribution surface to the theoretical value at certain time lag.

The model specified the conditional distribution at any time of the travel time distribution given the initial value is gaussian with calculated mean and variance. However we find this is different from the observations we get, as shown in the Los Angeles data [5]. A possible improved model may be
\[ X_t = \sum_i 1_{Y_t \in I_i} r^i_t \]
where
\[ I_i(t) \] is a partition on \([0, 1]\)
\[ Y(t) \sim U[0, 1] \]
\[ r^i_t \] satisfies: \( dr^i_t = a^i_t(b^i_t - r^i_t) dt + \sigma_t dW^i_t \)
Then the process is a randomization of different O-U processes. It is no longer a Gaussian process anymore and the properties of \( Y(t) \) should be further specified to achieve a reasonable solution. Interesting research can be conducted to tackle these
kind of pricing problem for non-normal travel time processes in future if the market is setup.

6 Conclusion

In this paper, we propose the new financial derivative-travel time derivative. It is a promising trading tool for the financial industry and ideal hedging and financing tool for the transportation industry. According to the market analysis, there are abundant participant in this emerging market and hedging, speculation can all be conducted which makes the it a fully-functioned financial tool. Possible products of travel time derivative are designed, including travel time index, options on travel time, options on congestion days and travel time default obligation. Since travel time derivative is a derivative based on the non-tradable underlying asset, to price it directly according to no arbitrage principle, we need to specify a market value of risk. Other method including indifferent pricing and hedging with partially correlated existing asset may also applicable. Interesting research can be conducted when both the market aspect and policy aspect agrees on the new area. As a innovative design, we propose it for discussion and opinions.

References


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Figure 2: The travel time derivative market
Figure 3: Travelers and travel time (QOS) protection
Figure 4: Alternative risk transfer between transportation related industry and financial industry

Figure 5: Conditional distribution surface of delay in Los Angeles