Using Historical Information in Forecasting Travel Times

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When solving the roadway network problem of finding the quickest route from point A to B, one would like to have the best information available on the travel times between locations in the network. Current systems find this shortest path based on hypothetical travel times calculated from distance and assumed speed. This paper develops a better solution through the creation of forecasts of travel times using historical and real-time information. This paper first develops a ten-parameter function, solely on historical information, using data from the Milwaukee Highway System. This function is the sum of three normal curves (each representing a “rush-hour” period) and a constant, fitted through a least-squares regression. Then, using a process developed through double-exponential smoothing, real-time information is added to the historical approximation. The use of this forecasting procedure in updating travel times in a network is then explained.
INTRODUCTION
When solving the roadway network problem of finding the quickest route between two locations, one would like to have the best information available on the travel times between these points in the network. One way to approximate these current travel times is to use historical travel times information to forecast current road conditions; traffic patterns based on time, such as time of day, for example, are easy to discern. These predictions are constructed from many individual observations in the past. If similar observations are received in real-time, the accuracy of predictions for future travel times should go up, especially in the short term. It is the goal of this paper to describe a process to forecast travel times, combining real-time information with historical.

Section 1 develops a method of predicting travel times using historical information and applies this to the Milwaukee Highway System. Section 2 defines real-time travel times. Section 3 introduces the concepts necessary to short-term travel time forecasting and applies these concepts to the Milwaukee Highway System. Section 4 briefly relates the results of this study to actual travel time prediction.

SECTION 1—HISTORICAL INFORMATION IN FORECASTING TRAVEL TIMES
The goal in this section is to create forecasts of travel times using historical information. These forecasts will serve as a base upon which to build a model that includes real-time information. In this section, representation and cleaning of historical travel time data is explored. Then, a methodology for developing an estimate of travel times from historical data is established. This step involves examining applicable data, investigating time categories, and developing a parameterized function as an estimate of travel time. Data from the Milwaukee Highway System is used to demonstrate our developed methodology.

1.1—Definition and Examination of Historical Travel Time
A historical travel time must include the following four characteristics: a beginning (point A) and end point (point B), a route between these points, the time to traverse this route, and the time in history that the observation occurred. Note that a travel time can not be assigned to a single instant; for the remainder of this paper, the convention of assigning a reference time to each observation of when the vehicle passed point B is followed. This reference time is referred to as the timestamp of the observation. Only at this time is all the necessary information in describing a travel time known. Thus, if an observation of 37 seconds for the travel time from A to B was recorded at 12:00:00, some measure was taken that indicates someone passing A at 11:59:23 would then pass B at 12:00:00. Therefore, a historical travel time is defined as a data point that can be received or constructed, that reflects the time it takes to transverse a specific route from one location to another ending at a given time in the past.

In doing this study, a travel time data set that includes all the above elements was desired. One such data set was received from John Mishefske at the Milwaukee Department of Transportation via email. (John Mishefske, unpublished data) Data was for seventeen A to B paths collected at inconsistent time intervals every several minutes for the entire month of June 2002 and July 6, 2002. Links included segments from intersection to intersection of major highways as well as from major exits to intersections. The data included endpoints, path, time stamp, and travel time. In all, 224,621 data points were received. The route from A to B is assumed to be the optimal path between the two points. The data set was cleaned, removing data.
points if any one of start or end points, time stamp, or travel time did not exist. Points were also removed if no route could be implied from the original data sources.

1.2—Functional Representation of Historical Travel Time

This acceptable data set of historical travel times can now be used to create a model estimating future travel times. Though many time elements could be considered, time of day is chosen as the most important predictor of travel time. The approach, developed through empirical observation and analysis of the Milwaukee Highway System, produces a continuous parameterized function of travel times as a function of time of day.

First of all, it takes some amount of time to travel a road link, even if there is little to no traffic. This minimum travel time is referred to as the free-flow travel time. In addition to this, the Milwaukee data shows that travel times are higher in the morning and afternoon, consistent with an expectation that morning and afternoon rush hours to and from work exist. The morning bump tends to have a symmetric bell shape; travel times during this morning peak period may be normally distributed around some mean. In evaluation of the Milwaukee Highway System travel times, afternoon peak travel times appear to show the same bell shaped curve, but with an extra hump before travel time reaches its maximum expected value. Empirically, it is conjectured that this is an additional set of trips dependent on afternoon activities such as the recess of schools and errands and can be represented by a third bell curve. Thus, the conjecture is that there are three bell curves, each representing one of the sets of trips described above, which correspond to travel congestion throughout each day. Each curve is multiplied by a coefficient and added together along with a constant for a minimum free flow travel time. Each normal curve has two parameters plus a multiplying factor; thus the function has ten parameters. This ten-parameter function is a continuous estimate of travel time as a function of time of day; this function is presented in Equation 1.

\[
\text{Weekday Travel Time} \\
TT = f(t) = K + C_1 \cdot \eta(\mu_1, \sigma_1) + C_2 \cdot \eta(\mu_2, \sigma_2) + C_3 \cdot \eta(\mu_3, \sigma_3) \\
\]

\[
\text{Where:} \\
\eta(\mu, \sigma) = \frac{1}{\sqrt{2\pi}\sigma^2} e^{-(t-\mu)^2/2\sigma^2} \\
\text{and } K, C_i, \mu_i, \sigma_i \text{ are parameters to be estimated}
\]

To choose the parameters for this function, this function is fitted to the data points themselves. Microsoft Excel was programmed to calculate the ten-parameter function for any time indicator while allowing freedom to choose the ten parameters. Excel Solver was run, allowing each of the parameters to vary, and minimizing the Sum of Squared Errors between the ten-parameter function and the observed data. Figure 1 displays one of the solutions found. In this figure each of the three normal curves are shown originating at zero. Above them is their sum added to the free flow value; also included as a comparison is the actual data.

This process was performed for each of the fifteen links in the Milwaukee data set. Results were consistent with the assumption of three normal curves representing the three

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groupings of high roadway usage. In general, afternoon peak periods tended to last a longer period of time and in many cases had a higher estimation of travel time. Morning peak periods were shorter in length and peaked more quickly.

The function presented above is one example of using historical information to predict current travel times. However, real-time information could also be added to update this function, hopefully improving predicted times. The rest of this paper deals with this problem.

SECTION 2—DEFINING REAL-TIME TRAVEL TIMES
A real-time travel time must contain all the same information as a historical travel time. Two locations must be specified: a start point (A) and an endpoint (B). Also, a route must be indicated between A and B. As mentioned in Section 1, it may be inferred that this path is the optimal path between the two points. The travel time is then the amount of time taken to go from A to B along the given path. The definition of historical travel times also included a time stamp indicating when the observation took place. The convention that the time stamp referred to the time the vehicle passes point B is needed, as that is the only time when all necessary components of the definition is known. For real-time travel times this convention is again adopted. By real-time, we mean now. Thus, for a travel time observation of t seconds from A to B received now to be a real-time observation, it must indicate that a vehicle passing A t seconds ago would currently be passing B. Thus, a real-time travel time is a data point that can be received or constructed and measures the time it takes to traverse a specific route from one location to another location ending now.

Note that instants after a real-time travel time observation is received, it becomes a historical travel time. Often, the “most recent real-time observation” is referred to. These observations are generally treated as if they are in real-time even if they are received a short while ago because they provide the most relevant information to the actual conditions of a roadway. In the following section, one possible process is described through which real-time travel times can be used to forecast travel times into the short-term future.

SECTION 3—REAL-TIME TRAVEL TIMES IN THE REAL WORLD
In order to produce usable information for a decision maker, an estimate of travel time for the seconds during which a driver will be actually passing from A to B must be produced. In Section 1, a solution to estimate travel time as a function of time of day was developed. This approach is static in nature and based strictly on observations of travel time in the past. In order to be more accurate in the prediction, inclusion of more information is desired. If a real-time information source is available and collected, a real-time measurement can be used, in addition to recent observations and a historical estimate, to base a forecast of travel time.

In this section, the combination of these information types on any path is used to predict future travel times along that same path. Sections 3.1 and 3.2 present two concepts and how they are necessary to this process, and Section 3.3 describes their use in forecasting travel times for the Milwaukee Highway System.

3.1—Exponential Smoothing and Travel Time Forecasting
Exponential smoothing is a method of “smoothing” a time series of observations. In this method, the most recent observations are given a high weight and previous observations are given lower weights that decrease exponentially with the age of the observation. This is where the name exponential smoothing comes from. Nearly all time series have an error or variance

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associated with their observation. By weighting past events in this way, this method creates a smoother picture of the time series without the noise associated with observation.

Two common types of exponential smoothing are single and double exponential smoothing. Single exponential smoothing weights an observation at time \( t-1 \) with the current smoothed estimate to get an updated estimate for the value of the time series at time \( t \). This method is equivalent to an ARIMA (0,1,1) time series model and is given by Equation 2.

\[
S_t = \alpha y_{t-1} + (1-\alpha) S_{t-1} \quad 0 \leq \alpha \leq 1 \quad t \geq 3
\]  

Here, \( S_t \) is the smoothed estimate and \( y_t \) is an observed value of a given time series. A smoothing factor \( \alpha \) is included to weight previous observations. By this formula, \( y_{t-1} \) is weighted by \( \alpha \), \( y_{t-2} \) is weighted by \( \alpha(1-\alpha) \), \( y_{t-3} \) is weighted by \( \alpha(1-\alpha)^2 \), and so on. The smoothing factor can be a constant or a direct function of \( t \) such as \( 1/t \) so long as it is between zero and one and should be estimated because different values for \( \alpha \) will produce different smoothing effects.

Double exponential smoothing adds a trend component to the single exponential smoothing concept. In this approach, an observation at \( t-1 \) is weighted with a previous estimate plus a trend factor, or estimated slope. This slope is also generally a smoothed estimate of the overall trend of the time series. As such, double exponential smoothing requires two smoothing parameters, given in Equation 3 as \( \alpha \) and \( \gamma \).

\[
S_t = \alpha y_{t-1} + (1-\alpha)(S_{t-1} + b_{t-1}) \quad 0 \leq \alpha \leq 1
\]

\[
b_t = \gamma(S_t + S_{t-1}) + (1-\gamma)b_{t-1} \quad 0 \leq \gamma \leq 1
\]  

Here, \( S_t \) and \( y_t \) again refer to the smoothed estimate and observation, respectively, and \( b_t \) is the smoothed estimate of the trend of the time series at time \( t \). By adding an estimate of the trend to the previous estimate of the value of a time series observation, double exponential smoothing is able to fit a time series that has a trend better than single exponential smoothing, which will lag behind if the value of time series observations continually increases or decreases.

Exponential smoothing provides a good way of following time series observations. It can also enable the prediction of future events. If no observation is available at time \( t \), the only information that can be known is the smoothed estimate at time \( t \), and trend and seasonality estimates if they exist. Because of this, the forecast of single exponential smoothing will always converge to a single value. Any forecast using double exponential smoothing will be the smoothed estimate at the time of the last observation plus a time increment to be forecasted for.

Exponential smoothing and adaptations thereof have been used in modeling traffic events. Shbaklo et al. introduce exponential smoothing as an excellent model for traffic quantities. They note a study performed by Levin in which volume data was collected and represented using several different ARIMA models and several different time lags between smoothing. Levin concluded that exponential smoothing, or ARIMA (0,1,1), produced the most statistically significant volume forecasts of the ARIMA models examined. The amount of volume on a roadway directly affects travel time; speed is often calculated as a direct function of roadway volume. So then, travel time could be modeled as a direct function of volume on a roadway. Therefore, exponential smoothing should also provide statistically significant results when forecasting travel times into the short-term future.

Additionally, Shbaklo et al. mention an approach to travel time forecasting that involves both the concepts of exponential smoothing and a historical profile of travel time. This approach was developed by Hoffman and Janko and produces a forecast of future travel time not by

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weighting the most recent observations overall, but by weighting a recent observation with a historical smoothed profile for that time of day. \((3)\) That is, for a set number of time intervals throughout the day, a historical profile exists that is a smoothing of previous day’s observations. To create a forecast, Hoffman and Janko weight this historical profile against new information to create an updated forecast, essentially performing single exponential smoothing for each time interval throughout the day. Their method is given by Equation \(4. (3)\)

\[
\overline{t}_{i,n}^{\text{new}} = \theta \overline{t}_{i,n}^{\text{old}} + (1-\theta) \overline{t}_{i,n}^{\text{old}}
\]

where:

- \(\overline{t}_{i,n}^{\text{new}}\) = new travel time value of the std. profile for link \(i\) during time interval \(n\)
- \(\overline{t}_{i,n}^{\text{old}}\) = previous existing travel time for link \(i\) during time interval \(n\)
- \(\overline{t}_{i,n}\) = corresponding travel time during most recent observation
- \(\theta\) = Weighting factor for the most recent observation

### 3.2—Recurring Traffic Patterns: Peaks and Free-flow

Also important to the goal of forecasting travel times in the short term is an understanding of how travel times are likely to behave throughout any given day. Several concepts are presented here so that they may be incorporated into the prediction process.

As noted in Section 1, traffic patterns will have peak periods where the observed travel times are more likely to be above the free flow travel time. Going one step further, note that on a given day, current travel times are highly correlated to recent travel times; this is why Shbaklo can say that ARIMA \((0,1,1)\) works. Because of this, on any given day, if travel times are high during a peak period, they will remain high. If travel times are low during a peak period, they will remain low.

However, because of the natural cause of travel times, a mean time at which people use the roadway (with some variance) should exist. As such, a time series of travel times will likely exhibit the overall pattern of the historical model of peak periods. That is, travel times should increase to a maximum point and then decrease after the maximum volume has occurred on the roadway. Overall, travel times are expected to mirror the general trend of the historical function developed in Section 1, and to mirror that trend either completely above or below the function. Note that the real traffic data generally follows the historical trend (shape) of morning and afternoon high travel times. However, the actual travel times for these peaks are either, for each peak, all higher or all lower, than the predicted curve. Figure 2 shows this empirically.

In addition to these “rush-hour” peaks, another important dynamic is apparent. There should be a time at which enough vehicles traveling during peak travel periods will reach their destinations to sufficiently lower traffic volume to that of non-peak periods. With little to no traffic, travel times should be approximately at free-flow levels. With the above description, this is likely to occur after each peak period. Realistically, in many local road networks, there will often be a daytime travel volume that is above the free flow limit. In this case, travel times should still drop off significantly after the morning peak. Except for rare occasions, nighttime travel times are expected to return to free flow in the absence of an event.

Fortunately, in the Milwaukee Highway System, this “return to normalcy” is inherent in our ten-parameter function modeling historical travel time. In Figure 2, the pink link represents

this function for one particular link. Overlaid onto this function are the actual recorded travel times for one day in June 2002. The return to normalcy is clear in the actual data. After each peak period ended, the travel times along this road segment quickly returned to free flow. Data showed similar trends during almost all days on all segments for which data was obtained.

Additionally, note the pattern of travel times around peak periods in Figure 2. In each case, data mirrors the pink historical travel time function, residing at a relatively constant proximity to it. In other words, if travel times are high for a given peak period, they will remain high throughout that peak period. Data was similar for nearly every day on every link examined.

Thus, travel times will mirror the historical perception of travel times, exhibiting morning and afternoon peaks as well as a return to free flow conditions outside of peak hours. Note that while mirroring the general shape of the historical prediction, actual travel time data is either all higher or all lower than these historical expectations during peak times. In the next section modeling these realities and predicting them in the Milwaukee Highway System using a variation of exponential smoothing is explored.

3.3—Application to Milwaukee Highway System
As already noted, the Milwaukee Highway System exhibits the properties described in Section 2. These properties of travel times are inherently different for different times of the day, and the model must be able to include these dissimilarities.

During peak hours, travel times are expected to mirror the shape of the historical distribution. For this to happen, any smoothing and forecast should reflect the slope of the historical distribution as a trend. The next smoothing and subsequent forecasts should also rely upon the most recent real-time observation and other recent observations in decreasing order of importance. This leads to the use of a concept similar to double exponential smoothing. Double exponential smoothing allows for a trend factor to be added on to any smoothed estimate and smoothes that trend factor. In this case, the trend it already known; it is the slope of the historical distribution function.

Therefore, during peak periods, a smoothed estimate of travel time is created by weighting the most recent real-time observation with the most recent smoothed estimate plus the difference in the historical times corresponding to the time of the previous smoothed estimate and the time associated with the current estimate. A forecast of the next period’s travel time would then just be the previous smoothed estimate plus the difference between the historical travel time function at the time of the smoothed estimate and forecast. This was implemented for the Milwaukee data in Microsoft Excel using Equation 5.
This concept will hold true during morning and afternoon peak travel periods. However, a return to free flow travel times at non-peak periods exists. Thus, these times must be treated differently. We want the smoothed estimates of travel time to both decay to free flow travel times and to reflect the most recent real-time observation. For this reason, when working with the Milwaukee data, the most recent real-time observation is not directly smoothed with the previous smoothed estimate and the free flow travel time. This is again essentially equivalent to double exponential smoothing in that the weighted average of a previous smoothed estimate and free flow travel time can be written as the smoothing term minus a trend term.

In forecasting during non-peak periods, the dependence on a real-time observation is removed and the most recent smoothing is weighted with the free flow travel time for each link. By the iterative nature of this process, an exponential decay to the free flow travel times is observed, as weights on previous smoothing decay exponentially. Equation 6 describes this process.

Note that different weighting parameters were used in $c$ and $\gamma$. This was done because the time intervals of real-time observations and forecasts might be necessarily different. This extra parameter allows flexibility in choosing a time increment for the iterative process of forecasting travel times.

These two circumstances were combined and implemented in Microsoft Excel in an iterative process. Real-time information could be received from Milwaukee via the internet or simulated by selecting a random day of existing historical data and progressing through that day observation by observation. In either case, data was parsed into 17 columns each corresponding

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to a beginning and endpoint pair. A timestamp was then associated with this vector noting the
time at which it was received. This information was then placed into a “Real-Time Information
Sheet” in Microsoft Excel. The worksheet could handle as many recent observations as
necessary, so long as they were placed in successive rows and the row following the most recent
real-time observation was left blank. A section of this worksheet is included in Figure 4. This
figure diagrams the process by which smoothed estimates of travel time and forecasts were
made.

In another worksheet, parameters were input. \( \theta \), \( \varphi \), \( \zeta \), and \( c \) (double-exponential
smoothing parameters) were included and allowed to range between 0 and 1. An additional
parameter was included for the time increment at which forecasts were to be produced. In other
words, if the increment were set to 30 seconds, a forecast would be created for every thirty
second interval beginning with the interval immediately following the most recent real-time
observation.

Historical estimates of travel times were then created on each link for every value of \( t \) that
a real-time observation was received as well as each increment past the most recent real-time
observation over a relatively short time horizon. These estimates were created based upon the
ten-parameter function described in Section 1 and the actual parameters estimated for each link
of the highway network.

An iterative process was then begun in which smoothing and forecasting were performed
by the conditions described above in the following way. A Visual Basic macro written for Excel
stepped through a series of increasing times, beginning with the time at which an initial real-time
observation was received. Time steps were associated directly with the time of observations,
generally occurring about every three minutes, until the most recent real-time observation.
Beginning with the first time, if a real-time observation was available, a smoothed estimate was
created differing if \( t \) lay within a peak travel time period. If no real-time information was
available at \( t \), a forecast was made based upon whether \( t \) lay within a peak period. For the
Milwaukee Highway System, a peak period was characterized by a historical estimate of travel
time that exceeded 105% of free flow travel time. This process is described in Figure 3.

A final worksheet was added to calculate the amount recent observations were weighted
at the moment of making a first forecast. Weights were assigned by the process described in
Section 3.1 and differed according to whether or not an observation occurred during a peak
period.

Note that while all data was stored in the spreadsheet used for this process, the actual data
requirements of this process are fairly small. The only information that needs to be stored is the
most recent travel time observation for each link, the most recent smoothing for each link, a
timestamp for each, and parameters for historical travel time functions.

Figure 3 describes the process of travel time prediction. Figure 4 describes the functional
implementation of this process in Microsoft Excel. Arrows note flow of information. Times
Corresponding to real-time observations and used to calculate historical function values.
Historical Function Values, Real-time values, and Smoothing Parameters are used to calculate
Smoothed Estimates and Forecasts. The number of real-time observations and smoothing
parameters are used to calculate weights on previous real-time observations.

The results of this method were first tested empirically. Simulations were run by
stepping through individual days and graphically displaying the results. The forecasts appeared
to predict future travel times with some degree of accuracy. An example of this empirical testing
is shown in Figure 5. In this figure, the pink link represents the historical estimate of travel time.

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The yellow points represent actual observations of travel time, and the blue points indicate the smoothed estimate of travel times and that estimate forecasted into the future. Looking at this figure, the blue forecast is able to closely predict actual travel time.

Empirical testing provided reasonably encouraging support in the belief that this method will in fact work to predict travel times. As further evidence, a quantitative measure of performance was sought out. A least absolute deviation measure over some time horizon into the future was constructed and implemented.

The Visual Basic macro used to step through any given day was altered to collect a running total of the least absolute deviation between the predicted and actual travel times for 6, 15, 30, and 60 minute time horizons after all observations. Smoothing parameters were chosen empirically for this testing. A total least absolute deviation was kept for each link in the Milwaukee Highway System. These totals were then divided by the number of observations to get an average least absolute deviation for each link. As a final step, these totals were expressed as a percentage of free flow travel time so that they may be compared directly and an overall measure of accuracy may be constructed for the entire system.

As a measure for comparison, average least absolute deviations were calculated for each time observation using the historical estimates of travel time as a predictor and using the free-flow travel time as a predictor.

This procedure was performed for each of the twenty weekdays in the month of June 2002 on all links in the Milwaukee Highway System. In general, the real-time prediction method outperformed the historical estimate and free-flow estimate of travel time. Shorter time horizons produced better results. These results are summarized in Figure 6 below.

Additionally, a system wide measure of accuracy was established for each method. This measure was an average least absolute deviation expressed as a percentage of free flow travel time. Again, the real-time prediction method outperformed the historical and free flow methods for all time horizons. As expected, shorter time horizons created more accurate predictions. This information is summarized in Figure 7.

Thus, given the correct smoothing parameters, the method of predicting travel time based on real-time information and a historical estimate will outperform the historical estimate and outperform a constant measure for travel time.

**SECTION 4—USING PREDICTED TRAVEL TIMES IN THE REAL WORLD**

The value of predicting travel time is to create a forecast of travel time that will be directly relevant to a decision maker and include the user’s decisions. A total travel time along a path form any start point to any endpoint is simply the sum of a continuous set of segments that connects the two points (plus time spent at intersections of road segments and any other considerations). Multiple paths generally exist between points of interest and the optimal path is considered to be that which minimizes the travel time of the decision maker. With real-time travel time information, costs assigned to roadways are more accurate and choices can be made that better optimize the value to the decision maker.

First, consider predicting travel time only using a function derived from historical data, for example, the function developed in Section 1. Suppose the historical estimate function of travel time from A to B is calculated to be \( \tau_{AB}(t) \). Let \( \Delta t \) be the current estimate of travel time. If a vehicle is at point A at time \( t \), the expected travel time to B is found by solving the simple equation:

\[ \Delta t = \tau_{AB}(t + \Delta t) \]

where:
- $\Delta t$ = current estimate of travel time
- $t$ = current time
- $\tau_{AB}()$ = historical estimate function of travel time from A to B

for $\Delta t$. This estimate of travel time, $\Delta t$, may then be assigned as the cost on a link at time $t$.

When incorporating real-time data, the key to forecasting link travel times is that the forecasts need to be for the time period when the user is expected to traverse the given link. Recall that a travel time measurement is associated with the end of an interval. Given this assumption, an approach similar to Equation 7 may be used to calculate travel time forecasts. However, Equation 7 is specific to a continuous estimation of historical travel time as a function of time of day. The forecasts, because they are calculated iteratively, are for discrete times, and the approach must be modified.

As a solution to this problem, one may either associate each moment one wants to predict with one of the discrete forecasts or one may look to somehow predict between the forecasts. An easy way to do this is simply by connecting all discrete points with a line. This link will represent any trend inherent in the discrete forecasts and produce a more accurate prediction of future travel time. The point where this continuous function of joined line segments satisfies Equation 7 is the desired predicted travel time. Equation 8 solves one instance of Equation 7 when adjacent discrete travel time forecasts are connected with a straight line.

\[
\Delta t = \frac{T(t_2)y_2 - T(t_2)y_1}{(T(t_1) + t_2) - (T(t_2) + t_1)}
\]

solves

\[
\Delta t = \left(\frac{T(t_2) - T(t_1)}{t_2 - t_1}\right)\Delta t + \left(\frac{T(t_1) - T(t_2)}{t_2 - t_1}\right)
\]

where:
- $\Delta t$ = forecasted travel time on a link beginning at an start - time
- $T(t)$ = forecast of travel time $t$ seconds from link start - time
- $t_1$ = closest time observation less than where equation 3.5 is satisfied
- $t_2$ = closest time observation greater than where equation 3.5 is satisfied

In this paper, a method of forecasting travel time was developed using historical and real-time data. First, a function based on three normal curves was calculated using only historical data. Then, using exponential smoothing, real-time data was added to this function, leading to a better forecast of travel time. The use of this forecasting procedure in updating travel times in a network was then explained. This technique or one similar to it will hopefully soon be applied to route guidance systems.

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REFERENCES

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FIGURE 1  Ten-parameter function fit to data from the Milwaukee Highway System. Function is summation of three bell curves and a constant.

FIGURE 2 Traffic patterns relative to a historical estimation of travel time for the Milwaukee Highway System.

Process repeated iteratively from first real time observation until some predetermined period after the most recent real-time observation.

\[
egin{align*}
\text{if} & \left [ \text{Real Time Information Received at } t \right ] \\
& \text{if } \left [ t \text{ During Peak Hour} \right ] \\
& S_n = \theta X_{n-1} + (1 - \theta) \left [ \tau(t_n) - \tau(t_{n-1}) + S_{n-1} \right ] \\
& \text{else} \\
& S_n = \phi X_{n-1} + (1 - \phi) \left [ \xi S_{n-1} + (1 - \xi) \tau(0) \right ] \\
& \text{end if} \\
& \text{else} \\
& \text{if } \left [ t \text{ During Peak Hour} \right ] \\
& S_n = \tau(t_n) - \tau(t_{n-1}) + S_{n-1} \\
& \text{else} \\
& S_n = c S_{n-1} + (1 - c) \tau(0) \\
& \text{end if} \\
& \text{end if} \\
\end{align*}
\]

where:
\[\tau(t_n) = 10 \text{ parameter function estimated in Chapter 3}\]
\[\{\theta, \phi, \xi, c\} \sim \text{smoothing parameters} \in \{0,1\}\]

FIGURE 3 Algorithm describing the smoothing and prediction of travel time observations.

Schrader, C. C., Kornhauser, A. L., Friese, L. M. Reacting in Real-Time, TRB 2004
FIGURE 4 Implementation of process of predicting travel times based on a smoothed estimates of real-time observations and a historical estimate of travel times. Arrows denote information flows.

Schrader, C. C., Kornhauser, A. L., Friese, L. M. Reacting in Real-Time, TRB 2004
FIGURE 5 Empirical testing of forecasting algorithm. (a) Forecast from most recent observation. (b) Weighting on most recent observations. (c) Realization of travel time data.

<table>
<thead>
<tr>
<th>Time Horizon of Prediction</th>
<th>Percent of Observations Better Than Historical Prediction</th>
</tr>
</thead>
<tbody>
<tr>
<td>6 min</td>
<td>95.9%</td>
</tr>
<tr>
<td>15 min</td>
<td>89.2%</td>
</tr>
<tr>
<td>30 min</td>
<td>78.7%</td>
</tr>
<tr>
<td>60 min</td>
<td>63.2%</td>
</tr>
</tbody>
</table>

FIGURE 6 Performance of real-time prediction method.

FIGURE 7 Average least absolute deviations of prediction methods expressed as a percentage of free-flow travel time.