Roughing it up:

Disentangling Continuous and Jump Components in Measuring, Modeling and Forecasting Asset Return Volatility

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Volatility Modeling and Forecasting in Financial Econometrics

- Asset pricing
- Portfolio choice
- Risk management
Realized Volatility

(...Merton....)

French, Schwert and Stambaugh (1987, JF)

Andersen, Bollerslev and Diebold

Barndorff-Nielsen and Shephard (2002, JRSS)

Fleming, Kirby and Ostdiek (2003, JFE)

Survey: Andersen et al. (2005), in Carey/Stulz (eds.),
Risks of Financial Institutions,
University of Chicago Press for NBER, forthcoming
Basic Theory

\[ dp(t) = \sigma(t) \, dW(t) \]

\[ r_{t,\Delta} = p(t) - p(t-\Delta) = \int_{t-\Delta}^{t} \sigma(s) \, dW(s) \]

\[ IV_t = \int_{t-\Delta}^{t} \sigma^2(s) \, ds \] (integrated vol, quadratic variation)

\[ RV_t = \sum_{j=1,\ldots,1/\Delta} r^2_{t-1+j,\Delta,\Delta} \] (realized vol, empirical quadratic variation)

\[ \lim_{\Delta \to 0} RV_t = IV_t \]

Simple Extensions: Drift, Multivariate, ...
Important Issues Remain Incompletely Resolved:

Separation of jump and diffusive movements
Why Care about Jumps, and Separating Jump from Diffusive Movements?

(1) Improved forecasts of RV for:

- Asset pricing
- Portfolio choice
- Risk management

(2) Improved understanding of the price discovery process and the macro/finance interface (e.g., Andersen, Bollerslev, Diebold and Vega, 2003 AER)
Jump Diffusion

\[ dp(t) = \mu(t) \, dt + \sigma(t) \, dW(t) + \kappa(t) \, dq(t) \]

Jump size \( \kappa(t) = p(t) - p(t-) \)

Counting Process \( q(t) \)

Time-varying intensity \( \lambda(t) \)

\[ P[dq(t) = 1] = \lambda(t)dt \]

Andersen, Benzoni and Lund (2002)
Chernov, Gallant, Ghysels, and Tauchen (2003)
Drost, Nijman and Werker (1998)
Maheu and McCurdy (2004), Pan (2002)
Jump Diffusion Integrated Volatility

\[ IV_{t+1} = \int_0^{t+1} \sigma^2(s) \, ds + \sum_{t \leq s \leq t+1} \kappa^2(s) \]

Jump Diffusion Realized Volatility

\[ RV_{t+1}^\Delta(\Delta) = \frac{1}{\Delta} \sum_{j=1}^{1/\Delta} r_{t+j\Delta,\Delta}^2 \rightarrow \int_0^{t+1} \sigma^2(s) \, ds + \sum_{t \leq s \leq t+1} \kappa^2(s) \]

Andersen, Bollerslev and Diebold (2004)
Andersen, Bollerslev and Diebold and Labys (2003)
Realized Bi-Power Variation
(The Key to Everything, I)

Barndorff-Nielsen and Shephard (2004a, 2005)

Builds on earlier work on realized power variation:
Aït-Sahalia (2003)
Barndorff-Nielsen and Shephard (2003a)

\[
BV_{t+1}(\Delta) \equiv \mu_1^{-2} \sum_{j=2}^{1/\Delta} \left| r_{t+j\cdot\Delta,\Delta} \right| \left| r_{t+(j-1)\cdot\Delta,\Delta} \right|
\]

\[
\rightarrow \int_t^{t+1} \sigma^2(s) ds
\]
So use:

$$J_{t+1}(\Delta) \equiv RV_{t+1}(\Delta) - BV_{t+1}(\Delta) \to \sum_{t \leq s \leq t+1} \kappa^2(s)$$
Data


30-Year T-bond futures, 1/1990 - 12/2002, 3,213 days (Chicago Board of Trade)
First-Pass Practical Implementation

Sampling Frequency $\Delta \to 0$

Five-Minute Returns

$\Delta = 1/288$ for DM/$$

$\Delta = 1/97$ for S&P and T-Bond

Single adjustment:

$$J_{t+1}(\Delta) = \max[RV_{t+1}(\Delta) - BV_{t+1}(\Delta), 0]$$
Figure 1B
Daily S&P500 Realized Volatilities and Jumps
Important Issues:

- Assessing jump “significance”
- Handling microstructure noise
Assessing Jumps I

Asymptotic (Δ→0) distribution in the absence of jumps:

\[
\Delta^{-1/2} \frac{RV_{t+1}(\Delta) - BV_{t+1}(\Delta)}{t+1} \left[ (\mu_1^{-4} + 2\mu_1^{-2} - 5) \int_{t}^{t+1} \sigma^4(s) \, ds \right]^{1/2} \to N(0, 1)
\]

Barndorff-Nielsen and Shephard (2004a, 2005)

Infeasible...
Realized Tri-Power Quarticity

\[ TQ_{t+1}(\Delta) = \Delta^{-1} \mu_{4/3}^{1/\Delta} \sum_{j=3}^{1/\Delta} |r_{t+j\Delta,\Delta}|^{4/3} |r_{t+(j-1)\Delta,\Delta}|^{4/3} |r_{t+(j-2)\Delta,\Delta}|^{4/3} \]

\[ \rightarrow \int_{t}^{t+1} \sigma^4(s) ds \]

Builds on earlier work on realized power variation:
Andersen, Bollerslev and Meddahi (2005)
$W_{t+1}(\Delta) \equiv \Delta^{-1/2} \frac{RV_{t+1}(\Delta) - BV_{t+1}(\Delta)}{\left[ (\mu_1^{-4} + 2\mu_1^{-2} - 5) TQ_{t+1}(\Delta) \right]^{1/2}}$

Feasible!
Incorporation of Variance Stabilizing Transforms and Maximum (Jensen’s Inequality) Adjustment

\[ Z_{t+1}(\Delta) = \Delta^{-1/2} \frac{[RV_{t+1}(\Delta) - BV_{t+1}(\Delta)]RV_{t+1}(\Delta)^{-1}}{[\mu_1^{-4} + 2\mu_1^{-2} - 5\max\{1, TQ_{t+1}(\Delta)BV_{t+1}(\Delta)^{-2}\}]^{1/2}} \]

Barndorff-Nielsen and Shephard (2004b)
Huang and Tauchen (2005)

Very well behaved in finite samples

Huang and Tauchen (2005)
A Concise Statement

\[ RV_{t+1}(\Delta) = C_{t+1,\alpha}(\Delta) + J_{t+1,\alpha}(\Delta) \]

\[ C_{t+1,\alpha}(\Delta) \equiv I[Z_{t+1}(\Delta) \leq \Phi_\alpha] \cdot RV_{t+1}(\Delta) + I[Z_{t+1}(\Delta) > \Phi_\alpha] \cdot BV_{t+1}(\Delta) \]

\[ J_{t+1,\alpha}(\Delta) \equiv I[Z_{t+1}(\Delta) > \Phi_\alpha] \cdot [RV_{t+1}(\Delta) - BV_{t+1}(\Delta)] \]

Depends on \( \alpha \) and \( \Delta \)

Previous \( J_{t+1} \) corresponds to \( \alpha = 0.5 \)
Theory and Practice

Theory: $\Delta \to 0$

Practice: Market Microstructure Frictions

Discreteness
Bid-Ask Bounce
Unevenly Spaced Observations

Aït-Sahalia, Mykland and Zhang (2005)
Andersen, Bollerslev, Diebold and Labys (2000)
Bandi and Russell (2004a,b)
Corsi, Zumbach, Müller, and Dacorogna (2001)
Curci and Corsi (2003), Hansen and Lunde (2005, 2004a,b)
Zhou (1996)
Returns Polluted by Microstructure Noise

\[ r_{t,\Delta} = p^*(t) - p^*(t-\Delta) + \nu(t) - \nu(t-\Delta) \equiv r^*_{t,\Delta} + \eta_{t,\Delta} \]

\( p^*(t) \): “true” price

\( \nu(t) \): microstructure noise

\( \eta_{t,\Delta} \) is MA(1)
New Robust Estimators

“Standard” Bi-Power Variation

\[ BV_{t+1}(\Delta) = \mu_1^{-2} \sum_{j=2}^{1/\Delta} |r_{t+j\cdot\Delta,\Delta}| \bigg| |r_{t+(j-1)\cdot\Delta,\Delta}| \]

Staggered Bi-Power Variation

\[ BV_{1,t+1}(\Delta) = \mu_1^{-2} (1 - 2\Delta)^{-1} \sum_{j=3}^{1/\Delta} |r_{t+j\cdot\Delta,\Delta}| \bigg| |r_{t+(j-2)\cdot\Delta,\Delta}| \]
“Standard” Tri-Power Quarticity

\[ TQ_{t+1}(\Delta) = \Delta^{-1} \mu_{4/3}^{-3} \sum_{j=3}^{1/\Delta} |r_{t+j\Delta,\Delta}|^{4/3} \left| r_{t+(j-1)\Delta,\Delta} \right|^{4/3} \left| r_{t+(j-2)\Delta,\Delta} \right|^{4/3} \]

Staggered Tri-Power Quarticity

\[ TQ_{1,t+1} = \Delta^{-1} \mu_{4/3}^{-3} \left( 1 - 4\Delta \right)^{-1} \cdot \sum_{j=5}^{1/\Delta} \left| r_{t+j\Delta,\Delta} \right|^{4/3} \left| r_{t+(j-2)\Delta,\Delta} \right|^{4/3} \left| r_{t+(j-4)\Delta,\Delta} \right|^{4/3} \]
Results
Figure 1B
Daily S&P500 Realized Volatilities and Jumps
6/30/99: FED raised short rate by ¼ percent at 13:15 CST but indicated that it “might not raise rates again in the near term due to conflicting forces in the economy.”

7/24/02: Record NYSE trading volume of 2.77 billion shares
Return to a Key Motivational Issue:

How do jumps feed into subsequent volatility movements?
Heterogeneous AR Realized Volatility (HAR-RV) Model

Corsi (2003)

\[
RV_{t+1} = \beta_0 + \beta_D RV_t + \beta_W RV_{t-5,t} + \beta_M RV_{t-22,t} + \epsilon_{t+1}
\]

\[
RV_{t,t+h} = h^{-1}( RV_{t+1} + RV_{t+2} + ... + RV_{t+h} )
\]

\[ h = 1, 5, 22 \text{ (daily, weekly, monthly)} \]

Approximate long-memory model
HAR-RV-J Model

\[ RV_{t+1} = \beta_0 + \beta_D RV_t + \beta_W RV_{t-5,t} + \beta_M RV_{t-22,t} \]

\[ + \beta_J J_t + \epsilon_{t+1} \]
HAR-RV-CJ Model

\[ RV_{t+1} = \beta_0 + \beta_{CD} C_t + \beta_{CW} C_{t-5,t} + \beta_{CM} C_{t-22,t} \]

\[ + \beta_{JD} J_t + \beta_{JW} J_{t-5,t} + \beta_{JM} J_{t-22,t} + \varepsilon_{t+1} \]
### Daily, Weekly, and Monthly DM/$ HAR-RV-J Regressions

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<th>$RV_{t,t+h}$</th>
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<tr>
<td></td>
<td>$h = 1$</td>
<td>$h = 5$</td>
<td>$h = 22$</td>
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<tr>
<td>$\beta_0$</td>
<td>0.083</td>
<td>0.132</td>
<td>0.231</td>
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<tr>
<td></td>
<td>(0.015)</td>
<td>(0.018)</td>
<td>(0.025)</td>
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<tr>
<td>$\beta_D$</td>
<td>0.430</td>
<td>0.222</td>
<td>0.110</td>
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<tr>
<td></td>
<td>(0.043)</td>
<td>(0.040)</td>
<td>(0.022)</td>
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<tr>
<td>$\beta_W$</td>
<td>0.196</td>
<td>0.216</td>
<td>0.218</td>
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<tr>
<td></td>
<td>(0.063)</td>
<td>(0.055)</td>
<td>(0.043)</td>
</tr>
<tr>
<td>$\beta_M$</td>
<td>0.244</td>
<td>0.323</td>
<td>0.225</td>
</tr>
<tr>
<td></td>
<td>(0.061)</td>
<td>(0.068)</td>
<td>(0.062)</td>
</tr>
<tr>
<td>$\beta_J$</td>
<td>-0.486</td>
<td>-0.297</td>
<td>-0.166</td>
</tr>
<tr>
<td></td>
<td>(0.096)</td>
<td>(0.070)</td>
<td>(0.056)</td>
</tr>
</tbody>
</table>

$R^2_{HAR-RV}$ | 0.252 | 0.261 | 0.215 |

$R^2_{HAR-RV-J}$ | 0.364 | 0.417 | 0.353 |
<table>
<thead>
<tr>
<th></th>
<th>(RV_{t,t+h})</th>
<th>h = 1</th>
<th>h=5</th>
<th>h=22</th>
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<td>(\beta_0)</td>
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<td>0.083</td>
<td>0.131</td>
<td>0.231</td>
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<td></td>
<td></td>
<td>(0.015)</td>
<td>(0.018)</td>
<td>(0.025)</td>
</tr>
<tr>
<td>(\beta_{CD})</td>
<td></td>
<td>0.407</td>
<td>0.210</td>
<td>0.101</td>
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<tr>
<td></td>
<td></td>
<td>(0.044)</td>
<td>(0.040)</td>
<td>(0.021)</td>
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<tr>
<td>(\beta_{CW})</td>
<td></td>
<td>0.256</td>
<td>0.271</td>
<td>0.259</td>
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<td></td>
<td></td>
<td>(0.077)</td>
<td>(0.054)</td>
<td>(0.046)</td>
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<tr>
<td>(\beta_{CM})</td>
<td></td>
<td>0.226</td>
<td>0.308</td>
<td>0.217</td>
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<td></td>
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<td>(0.072)</td>
<td>(0.078)</td>
<td>(0.074)</td>
</tr>
<tr>
<td>(\beta_{JD})</td>
<td></td>
<td>0.096</td>
<td>0.006</td>
<td>-0.002</td>
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<tr>
<td></td>
<td></td>
<td>(0.089)</td>
<td>(0.040)</td>
<td>(0.017)</td>
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<tr>
<td>(\beta_{JW})</td>
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<td>-0.191</td>
<td>-0.179</td>
<td>-0.073</td>
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<td></td>
<td></td>
<td>(0.168)</td>
<td>(0.199)</td>
<td>(0.125)</td>
</tr>
<tr>
<td>(\beta_{JM})</td>
<td></td>
<td>-0.001</td>
<td>0.055</td>
<td>-0.014</td>
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<tr>
<td></td>
<td></td>
<td>(0.329)</td>
<td>(0.460)</td>
<td>(0.604)</td>
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</table>
Summary and Directions for Future Work

• Useful to split jump from diffusive phenomena

• We have developed a nonparametric framework for doing so
  • High-frequency data is key ingredient
  • Assessment of “significant” jumps
  • Robustness to microstructure noise
  • New associated HAR-RV-J and HAR-RV-CJ models produce superior forecasts

• Future: Dynamics of Jump Intensities and Sizes