Time for a Time-Change: A new Approach to Multivariate Intensity Models of Credit Risk

Philipp J. Schönbucher  D-MATH, ETH Zürich

Princeton, May 2008
Outline

1. Introduction
   - Pricing Single-Tranche CDOs

2. Current Multivariate Intensity Models

3. Time-Changed Intensity Models

4. Possible Specifications of Time Changes

5. Implementation
Current Portfolio Credit Risk Models

- Gauss copula (current standard): Widespread dissatisfaction
  - (i) Ad-hoc “fit” via base correlation / recovery curves
  - (ii) Unrealistic term-structure properties.
  - (iii) Instability of parameters
  - (iv) No proper dynamics, no consistent hedging.
  - (v) Various quick-fixes exist for (i) and (ii).

- Multivariate firm’s value models:
  - (i) Too little flexibility: Bad fit to single-obligors already
  - (ii) Numerically prohibitively intensive for portfolios
  - (iii) Plausible story, fundamental link.

- Top-down models:
  - (i) Excellent fit and dynamics for standard index portfolios
  - (ii) Aggregated, hard to get hedges against individual obligors

- Multivariate intensity models: Probably best way forward
  (more later)
Requirements from a New Model

**Application:**
- **Bespoke Tranches:** Extrapolation of structure from indices to other portfolios.
- **Exotic credit derivatives:** Forward-starting tranches, various options on tranches and index, Leveraged super-senior tranches.

**Hedging:**
- Realistic CDS (individual) and CDO (portfolio) dynamics.

**Calibration:**
- to single-obligor survival probability curves
- to CDO tranches on standard indices

**Numerical efficiency:**
- fast calibration
- which (almost) requires conditional independence
Related Literature

- Multivariate intensity models: Duffie and Garleanu [2001], Gaspar and Schmidt [2005], Mortensen [2005]
- Time-changes in credit risk:
  Joshi and Stacey [2005]: special case and precursor of this paper
  Giesecke and Tomecek [2005]: time-changes in top-down approaches
- Time-changes in option pricing: Clark [1973], Madan et al. [1998], Geman et al. [2001], Cont and Tankov [2004], Carr et al. [2003]
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Obligors and Loss Process

- $i = 1, \ldots, I$ obligors
- with default times $\tau_i$
- and default indicator processes $D_i(t) = 1_{\{\tau_i \leq t\}}$.

The key quantity of the model is the **default loss process**

$$L(t) := \sum_{i=1}^{I} D_i(t).$$

(losses given default are normalized to one)
A Typical Loss Process

Cumulative loss process $L_C$ of a STCDO with lower and upper attachment points $K_1$ and $K_2$

$$L_C(t) = (L(t) - K_1)^+ - (L(t) - K_2)^+. $$
## Market Quotes: STCDOs on iTraxx Europe

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Quotes for loss protection on tranches of European iTraxx Series 4, on Sept. 26th, 2005. Lower and upper attachment points are in % of notional, base correlation (BC) is given in %. Prices for the 0-3 tranche are % of notional upfront plus 500bp running, all other prices are bp p.a.. Source (including BCs): JPMorgan.
Remarks

- Liquid markets exist for STCDOs on standard indices.
- Standard pricing model: 1-Factor Gauss copula.
- Widespread dissatisfaction with performance and properties of Gauss copula models.
  - Inability to “fit”, problems when interpolating base correlations.
  - Instability of parameters (GM/Ford May 2005)
  - Unrealistic term-structure properties.
  - No proper dynamics, no consistent hedging.
- Exotic credit derivatives:
  Forward-starting tranches, options on tranches and index,
  Leveraged super-senior tranches.
The Cumulative Loss

- The initial cumulative loss is zero: $L(0) = 0$.
- At the $j$-th credit event $\tau(j)$ ($1 \leq j \leq I$), the cumulative loss is increased by the loss at this default:

$$dL(t) = \sum_{i=1}^{I} E_i (1 - R_i) dD_i(t).$$

$E_i$ exposure, and $R_i$ recovery rate of obligor $i$.
- The cumulative loss of the tranche $L^C(t)$ is the amount by which the cumulative loss of the portfolio has exceeded the lower bound $K_1$, capped at the upper bound $K_2$:

$$L^C(t) = (L(t) - K_1)^+ - (L(t) - K_2)^+.$$
Default and Fee Payment

The **default payment** of the protection seller to the protection buyer at a default event is the increase in the cumulative loss of the tranche:

\[ L^C(\tau_i) - L^C(\tau_i^-) \]

The protection buyer pays a periodic **protection fee** of \( \bar{s} \) of the remaining notional of the tranche.

\[ \bar{s} \cdot \left[ K_2 - K_1 - L^C(t) \right] \, dt. \]
Pricing Tranche Protection I

$L^C$ cumulative loss of the tranche:
Loss payment at time $t = \text{increment in } L^C \text{ at time } t$.
The NPV of the loss payments of the tranche can be transformed using integration-by-parts:

$$\int_0^T \beta(t) dL^C(t) = \beta(T)L^C(T) - \int_0^T L^C(t) d\beta(t)$$

$$= \beta(T)L^C(T) + \int_0^T L^C(t)\beta(t)r(t)\,dt.$$

- $\beta(t) = \exp\{-\int_0^t r(s)\,ds\}$ is the default-free discount factor.
- $d\beta(t) = -r(t)\beta(t)\,dt$.
- This holds for each sample path (and not just on average).
Pricing Tranche Protection II

Assume independence of defaults and default-free interest rates:

\[
E^Q \left[ \int_0^T \beta(t) dL^C(t) \right] = B(0, T) \ E^Q \left[ L^C(T) \right] \\
+ \int_0^T \ E^Q \left[ L^C(t) \right] f(0, t) B(0, t) dt, \\
\]

\[
f(0, t) = -\frac{\partial}{\partial T} \ln B(0, T) \text{ are the default free forward rates.}
\]

**Note:**

We only need the distribution or the density \( f^L(x, t) \) of the cumulative loss of the **whole** portfolio. The expected tranche loss for each tranche is then:

\[
E^Q \left[ L(t) \right] = \int_{K_1}^{K_2} (x - K_1) f^L(x, t) dx + (K_2 - K_1) F^L(K_2, t).
\]
The Fee Payment

The NPV of the reduction of the fee payment in one given scenario is

$$\int_0^T \bar{s} \cdot L^C(t) \beta(t) \, dt.$$ 

Its value is

$$\bar{s} \int_0^T \mathbf{E}^Q \left[ L^C(t) \beta(t) \right] \, dt = \bar{s} \int_0^T \mathbf{E}^Q \left[ L^C(t) \right] B(0,t) \, dt.$$ 

Again, only dependence on the value of $L^C(t)$ (or $L(t)$) at all times $t \leq T$. 
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Typical Setup of Multivariate Intensity Models

- Each obligor $i \leq I$ has a default arrival process $N_i(t)$ with intensity $\lambda_i(t)$.

- Conditional on the joint realisation of $\{\lambda_1(t), \lambda_2(t), \ldots, \lambda_I(t)\}_{t \geq 0}$, the $N_i(t)$ are indep. inhomog. Poisson processes with intensities $\lambda_i(t)$. (Joint Cox process)

- Individual default intensities $\lambda_i(t)$ are modeled as a (weighted) sum of:
  - a common stochastic factor $\lambda^G(t)$
  - and an independent idiosyncratic component $\lambda^{id}_i(t)$

$$\lambda_i(t) = w_i \lambda^G(t) + \lambda^{id}_i(t).$$
Additive Specification: General Remarks

\[ \lambda_i(t) = w_i \lambda^G_i(t) + \lambda^{id}_i(t). \]

- Interpretation as competing risks model.
- Intrinsic bounds on risks:
  - \( w_i \lambda^G_i(t) \) is the lowest possible level that \( \lambda_i \) can reach.
  - In large (homogeneous) portfolios:
    Portfolio default rate is always larger than \( \lambda^{id}_i(t) \).
- Initial Fit to Single-Name CDS
  High-quality obligors will need lower \( w_i \), will have (relatively) little systematic risk if downgraded
- Dynamics depend on quality of obligors.
- Specification: Strong co-movements of \( \lambda_i \) are necessary to reach realistic default dependence (i.e. high volatility of \( \lambda^G_i \)).
Additive Specification: Conditional Independence

\[ X(T) := \int_0^T \lambda^G(t) \, dt \quad \text{the common factor} \]
\[ P_x(T, b) := \mathbb{E} \left[ e^{-bX(T)} \right] \quad \text{the systematic part of the PS} \]
\[ P_i(T) = \mathbb{E} \left[ e^{-\int_0^T \lambda^{id}_i(t) \, dt} \right] \quad \text{the idiosyncratic part of the PS} \]

Then, \textbf{conditional} on \( X(T) \), survivals up to \( T \) are independent with individual conditional survival probabilities

\[ P_{Xi}(T, X(T)) = e^{-w_iX(T)} P_i(T) \]

The \textbf{unconditional} survival probability of obligor \( i \) until time \( T \) is:

\[ \mathbb{P} \left[ \tau_i > T \right] = P_x(T, w_i) P_i(T) \]
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Modelling Strategy

1 Specify a benchmark, **pre** time-change model \((F_t)_{t \geq 0}\): Tractable, easily understood, defaults are independent.

2 Define a time-change \(T\).
   The real-world time \(t\) is mapped to the (random) time \(T_t\) in the pre time-change model.

3 Result: The real-world, **post** time-change model \((G_t)_{t \geq 0}\): realistic, defaults are dependent.
Pre Time-Change Model

For each obligor $i = 1, \ldots, I$, we have

(i) an $(\mathcal{F}_t)_{t \geq 0}$-adapted, pre time-change intensity $\tilde{\lambda}_i(s) \geq 0$,

(ii) a unit exponentially distributed default trigger variable $E_i$.

$\tilde{\lambda}_i(s)$ and $E_i$ are independent from each other and across obligors. The pre time-change default time of obligor $i$ is:

$$\tilde{\tau}_i = \inf\{t \geq 0 \mid \int_0^t \tilde{\lambda}_i(s) \, ds \geq E_i\},$$
Remarks

- $\tilde{\lambda}_i(t)$ will not be the intensity after the time change. The “true” intensity is given later.
- No big loss of generality to assume $\tilde{\lambda}_i(t)$ non-stochastic. Idiosyncratic dynamics of the intensities do not affect the prices of CDS or STCDOs.
- The values of the pre time-change intensities will have to be found from marginal survival probabilities $P_i(0, T)$ by calibration.
- **Conditional independence:** Conditional on the full path of the time change $T_t$, defaults are independent from each other.
- But also: Conditional on $(\mathcal{F}_t)_{t\geq 0}$, $N_i(t)$ are independent.
Stochastic Time Changes I

- A *time change* is a right-continuous, increasing, $[0, \infty]$-valued stochastic process $(T_s)_{s \in \mathbb{R}_+}$ such that $T_s$ is a $(\mathcal{F}_t)_{t \geq 0}$-stopping time for any $s \in \mathbb{R}_+$.

- The distribution and density functions of $T$:

  \[ F(t, s) := \mathbb{P}[T_t \leq s] \]

  \[ f(t, s) := \frac{\partial}{\partial s} F(t, s). \]

  One may normalise the mean of $T_t$ to $\mathbb{E}[T_t] = t$, this should add stability to the calibration procedure.
Stochastic Time Changes II

By \((\mathcal{G}_t)_{t \geq 0}\) we denote the *time-changed filtration*

\[
\mathcal{G}_s := \mathcal{F}_{T_s},
\]

where \(\mathcal{F}_{T_s}\) is the sigma algebra of all events observable up to the stopping time \(T_s\).

\((\mathcal{G}_t)_{t \geq 0}\) is increasing (it is still a filtration), right-continuous (if \(T\) is right-continuous), and complete, thus \((\mathcal{G}_t)_{t \geq 0}\) is indeed a proper filtration which satisfies the usual conditions.
Time-Changed Processes

Let $X$ be a process adapted to $(\mathcal{F}_t)_{t \geq 0}$, and let $T$ be a finite time change. The time-changed process $X^T$ is defined as

$$X^T(s) := X(T_s)$$

$X^T$ is $(\mathcal{G}_t)_{t \geq 0}$-adapted.

If $X$ is $(\mathcal{F}_t)_{t \geq 0}$-independent from $T$, then for every $t \geq 0$ we have

$$\mathbb{E} \left[ X^T(t) \right] = \int_0^\infty \mathbb{E} \left[ X(s) \right] f(t, s) ds.$$ 

Furthermore, for $t > u \geq 0$,

$$\mathbb{E} \left[ X^T(t) \mid \mathcal{G}_u \right] = \int_{T(u)}^\infty \mathbb{E} \left[ X(s) \mid \mathcal{G}_u \right] f_u(t, s) ds.$$
The Post Time-Change Model

For each obligor $i = 1, \ldots, I$, the post time-changed default time is

$$\tau_i = \inf\{t \geq 0 \mid \int_0^{T(t)} \tilde{\lambda}_i(s) ds \geq E_i\}.$$

or equivalently,

$$\tau_i = \inf\{t \geq 0 \mid \tilde{\tau}_i \leq T(t)\}.$$

The filtration of the post time-change model is $(\mathcal{G}_t)_{t\geq0}$. 
Survival Probabilities Post Time-Change

Integrate over all possible realisations of the time change:

\[
P_i(0, t) = \mathbb{P} [ \tau_i > t ] = \mathbb{E} \left[ \mathbb{P} [ \tau_i > t \mid T_t = s ] \right]
\]

\[
= \int_0^\infty e^{-\int_0^s \tilde{\lambda}_i(u) du} f(t, s) ds.
\]

For constant \( \tilde{\lambda}_i \), the individual survival probabilities are

\[
P_i(0, t) = \mathbb{E} \left[ \exp\{-\tilde{\lambda}_i T_t\} \right] =: \mathcal{L}_t(\tilde{\lambda}_i),
\]

where \( \mathcal{L}_t(c) = \mathbb{E} \left[ e^{-cT(t)} \right] \) denotes the *Laplace transform* of \( T(t) \) for \( c \geq 0 \).
Cumulative Portfolio Loss Post Time-Change

Let $L(t) := \sum_{i=i}^{T} N_i(t)$. Then

$$\mathbb{P}\left[ L^T(t) \leq x \right] = \mathbb{E}\left[ \mathbb{P}\left[ L(T(s)) \leq x \mid T_t = s \right] \right]$$

$$= \int_0^{\infty} F_L(x, s) f(t, s) ds,$$

where $F_L(x, s) = \mathbb{P}\left[ L(s) \leq x \right]$ is the distribution of the pre time-change portfolio loss at time $s$.

- $F_L(x, s)$ can be found by semi-analytic convolution techniques.
- For different time changes and different post time-change reference points $t$, only the density $f(t, s)$ is different, the $F_L(x, s)$ remain the same.
Post Time-Change Intensities

If we can write $T$ as

$$T_t = \int_0^t \alpha(s)ds$$

for some stochastic process $\alpha$, then the default intensity $\lambda(t)$ of an obligor is given by:

$$\lambda_i(t) = \tilde{\lambda}_i(T_t)\alpha(T_t) = \tilde{\lambda}_i^T(t)\alpha^T(t).$$

It $T$ has a jump of $\Delta = T(t) - T(t-)$, there can be simultaneous defaults of several obligors with positive probability, and the local survival probability of a given obligor $i \leq I$ is equal to

$$\exp\{-\int_{T_t-}^{T_t+\Delta} \lambda_i(s)ds\}. $$
Qualitative Properties

\( \alpha \gg 0 \) Fast clock: (e.g. 1 post year \( \approx \) 10 pre years)

**Recession:** Many more events than average, more volatility, defaults cluster.

\( \alpha \approx 0 \) Slow clock: (e.g. 1 post year \( \approx \) 1 pre day)

**Boon:** Fewer events, low volatility, clustering of survivals.

- Realistic default dependence.
- Realistic connection between volatility and default rate.
- No lower bounds on default rates or conditional PDs.
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**Intensity-Gamma (IG) Time Change**

In the IG model (Joshi and Stacey [2005]), the time change is a Gamma process with mean $\mu = 1$. Thus, $T(t) \sim G\left(\frac{t}{\nu}; \nu\right)$. Its density is:

$$f^{IG}(t, s; \nu) = \frac{1}{\nu t^{\nu} \Gamma\left(\frac{t}{\nu}\right)} s^{\frac{t}{\nu} - 1} \exp\left\{-\frac{s}{\nu}\right\}.$$ 

- Gamma processes have i.i.d. increments and possibly large jumps.
- Dependence arises from joint defaults at the jump times of $T$.
- I.i.d. increments imply constant spreads.
- JS also consider sums of Gamma processes
- JS use Monte-Carlo to solve numerically (inefficient)
Frailty Time Changes

\[ T(t) = t \cdot Y \]

where \( Y \) is a nonnegative random variable with distribution \( F_Y(y) \), density \( f_Y(y) \) (if it exists), and \( \mathbb{E}[Y] = 1 \).

\[ F(t, s) = \mathbb{P}[T(t) \leq s] = \mathbb{P}[Y \leq s/t] = F_Y(s/t), \]
\[ f(t, s) = \frac{1}{t} f_Y(s/t). \]

- Examples: Gamma distribution (Clayton copula), discrete, lognormal ("Cox proportional hazards model")
- If we use on the post time-change model a filtration that is generated only by the default events, then we will have information-based default contagion. (That filtration is smaller than \( (\mathcal{G}_t)_{t \geq 0} \).)
- Can be extended to (over time) piecewise-constant \( Y \).
Continuous Stochastic Time Changes

\[ T_t := \int_0^t \alpha(s) ds. \]

- For single-obligor risk, any intensity-based model can be represented with a time-change (just choose \( \tilde{\lambda}_i = \text{const} \) and specify a suitable \( \alpha(t) \)).
- Need large volatility in \( \alpha \), e.g. jumps in \( \alpha \) to fit senior tranches:
  - Shot-noise process
  - Exponentially distributed jumps, triggered by Poisson process (e.g. Mortensen [2005])
  - Affine jump-diffusion processes.

In these cases, the Laplace transform of the distribution of \( T_t \) is known.
Weighted Sums of Time Changes

A *weighted sum* of time changes is achieved by setting

$$T(t) = \sum_{z=1}^{Z} w_z T_z(t)$$

where $w_z \geq 0$ are the weights of the individual time changes. Density and/or distribution of $T$ by Fourier/Laplace inversion of:

$$E \left[ e^{-cT(t)} \right] = \prod_{z=1}^{Z} E \left[ e^{-cw_zT_z(t)} \right].$$
Mixtures of Time Changes

A mixture of $Z$ time changes $T_1, \ldots, T_Z$ with mixing probabilities $p_z, z \leq Z$ is reached using a discrete random variable $X(\omega) \in \{1, \ldots, Z\}$ with distribution $\mathbb{P} [ X = z ] = p_z$ and setting

$$T(t) = T_X(t).$$

- The density (distribution) of $T(t)$ is simply the $p_z$-weighted average of the densities (distributions) of the $T_z(t)$.
- Event probabilities and prices of credit derivatives will become weighted averages of the respective prices conditional on $X = z$. 

Grouped Time Changes:

Let $T$ be a common time change and $T_g$ independent group-specific time changes (e.g. global vs. sector time-change). The individual $i$ from group $g(i)$ is time-changed with

$$T_i(t) := T_g(i) (T(t)).$$

The common time-change $T(t)$ must be performed before the groupwise idiosyncratic time change $T_g(\cdot)$, otherwise some groups will be able to look into the future.

The density of $T_i(t)$ is reached by convolution:

$$f_i(t, u) = \int_0^\infty f(t, s) f_g(s, u) ds.$$
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General Remarks On Implementation

- Conditional independence.
- Stochastic dynamics of $\lambda_i$ can be added after calibration of the model to the marginal $P_i$.
- Iterative calibration (similar to other intensity models) is possible: Calibrate STCDO and CDS curves separately and iteratively.
- Re-using time points makes STCDO pricing highly efficient.
- Need to integrate over distribution of $T_t$. 
STCDO Pricing Problem

STCDO pricing requires at all times $t \leq T$ before maturity knowledge of the distribution $F^L$ of the cumulative portfolio loss $L$

$$F^L(t, x) = \mathbf{P} [ L(t) \leq x ]$$

We denote the conditional loss distribution with

$$F^{CY}_{n} (t_k, \cdot) = \mathbf{P} [ L(t) \leq x \mid Y = y_n ]$$

If defaults are independent, the conditional portfolio loss distribution can be found in $O(I^2)$ operations using standard recursive algorithms.
Typical Numerical Efforts

In a standard factor model (e.g. Gauss copula), unconditional loss distributions are found with the following algorithm:

- Discretize time \( t \in \{0 = t_0, \ldots, t_{K_t}\} \)
- Discretize the conditioning factor \( Y \in \{y_0, \ldots, y_{K_y}\} \), and its distribution with quadrature weights \( w^y_n \).
- Approximate

\[
F^L(t_k, \cdot) \approx \sum_{n=0}^{K_y} w^y_n F^{CY}_n (t_k, \cdot)
\]

Total effort: \( K_t \approx 40 - 80, I \approx 125, K_y \approx 80 \)

\[
K_y \cdot K_t \cdot O(I^2).
\]
Numerical Effort in a Time-Changed Intensity Model

- Discretize real-time $t \in \{0 = t_0, \ldots, t_{K_t}\}$
- Discretize pre time-change time $s \in \{0 = s_0, \ldots, s_{K_s}\}$
- Calculate the pre time-change loss distributions: $K_s \cdot O(I^2)$

$$F^{TL}(s_k, x) = P[L(s_k) \leq x]$$

- Integrate the loss distributions to post time-change loss distributions: $K_t \cdot K_s$

Total effort: $K_t \approx 40 - 80$, $I \approx 125$, $K_s \approx 3K_t$

$$K_s \cdot O(I^2) + K_t \cdot K_s$$

We gained one order of magnitude by re-using results from previous time points.
Calibration Equations

*Linear* problem for individual CDS:
Given time-change parameters $\theta$, find probabilities $P_i^C(s) := e^{-\Lambda_i(s)}$ s.t.

$$P_i(t_l) = \int_0^\infty e^{-\Lambda_i(s)} f(t, s; \theta) ds =: \int_0^\infty P_i^C(s) f(t, s; \theta) ds.$$  

*Re-weighting* problem for STCDOs: Given $P_i^C(s)$

- construct the pre-TC loss distribution $F^{TL}(s_k, x)$ (only *once*).
- ... and find parameters $\theta$ (iteratively) of the time-change such that STCDOs are priced correctly.

The pre-TC loss distribution ($K_s \cdot O(I^2)$ effort) only has to be calculated *once*, changing TC parameters amounts to re-weighting.
**Calibration**

1. **Initialization:**
   Choose initial TC parameters $\theta_0$, and set $m = 0$.

2. **Iteration:** $m \rightarrow m + 1$
   - Calibrate single-name survival probabilities, given $\theta_m$ (linear)
   - Construct new pre-TC loss distribution.
   - If error in STCDO pricing is small, **EXIT** calibration loop.
   - Else: Find $\theta_{m+1}$ which minimizes the STCDO pricing error
   - Repeat.

Fixed-point is a fully calibrated model.
Preliminary numerical studies indicate significantly quicker convergence than global optimization.
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Raquel Gaspar and Thorsten Schmidt. Quadratic models for portfolio credit risk with shot-noise effects. SSE/EFI working paper series in economics and finance, no 616, 2005.


