Hedging credit index tranches
Investigating versions of the standard model

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Subtle company introduction
A standard model for credit index tranches exists.

It is commonly acknowledged that the common model is flawed.

Most of the focus is on the static flaw: the failure to calibrate to all tranches on a single day with a single model parameter.

But these are liquid derivatives. Models are not used for absolute pricing, but for relative value and hedging.

We will focus on the dynamic flaws of the model.
Outline

1. Standard credit derivative products
2. Standard models, conventions and abuses
3. Data and calibration
4. Testing hedging strategies
5. Conclusions
Standard products

- Single-name credit default swaps
  - Contract written on a set of reference obligations issued by one firm
  - Protection seller compensates for losses (par less recovery) in the event of a default.
  - Protection buyer pays a periodic premium (spread) on the notional amount being protected.
  - Quoting is on fair spread, that is, spread that makes a contract have zero upfront value at inception.
Credit default swap indices (CDX, iTraxx)

- Contract is essentially a portfolio of (125, for our purposes) equally weighted CDS on a standard basket of firms.
- Protection seller compensates for losses (par less recovery) in the event of a default.
- Protection buyer pays a periodic premium (spread) on the remaining notional amount being protected.
- New contracts (series) are introduced every six months.
- Standardization of premium, basket, maturity has created significant liquidity.
- Quoting is on fair spread, with somewhat of a twist.
- Pricing depends only on the prices of the portfolio names ... almost.
Standard products

- Tranches on CDX
  - Protection seller compensates for losses on the index in excess of one level (the attachment point) and up to a second level (the detachment point).
  - For example, on the 3-7% tranche of the CDX, protection seller pays losses over 3% (attachment) and up to 7% (detachment).
  - Protection buyer pays an upfront amount (for most junior tranches) plus a periodic premium on the remaining amount being protected.
  - Standardization of attachment/detachment, indices, maturity.
  - Not strictly a derivative on the index, in that payoff does not reference the index price.
  - Pricing depends on the distribution of losses on the index, not just the expectation.

- Also, options on CDX, but we will not consider these.
CDX history
CDX tranche history
What do we want from a model?

Fit market prices, but why?

- Extrapolate, i.e. price more complex, but similar, structures
  - Non-standard attachment points
  - Customized baskets
- Hedge risk due to underlying
  - Dealers provide liquidity in tranches, but want to control exposure to underlyings.
  - Speculators want to make relative bets on tranches without a view on underlyings.
- Risk management
  - Aggregate credit exposures across many product types.
  - Recognize risk that is truly idiosyncratic.

For anything other than extrapolation, we care about how prices evolve in time, so we should look at the dynamics.
Standard pricing models—basic stuff

- Price for a tranche is the difference of
  - Expectation of discounted future premium payments, and
  - Expectation of discounted future losses.

- Boils down to the distribution of the loss process on the index portfolio, in particular things like

$$E \min\{(d - a), \max\{0, L_t - a\}\}.$$ 

- Suffices to specify the joint distribution of times to default $T_i$ for all names in the basket.

- CDS (or CDX) quotes imply the marginal distributions for time to default for individual names $F_i(t) = P\{T_i < t\}$. 
Standard pricing models—specific stuff

- Dependence structure is a Gaussian copula:
  - Let $Z_i$ be correlated standard Gaussian random variables.
  - Default times are given by $T_i = F_i^{-1} (\Phi(Z_i))$.
- Correlation structure is pairwise constant . . .

\[ Z_i = \sqrt{\rho} Z + \sqrt{1 - \rho} \varepsilon_i. \]

- For a single period, just count the number of $Z_i$ that fall below the default threshold $\alpha_i = \Phi^{-1}(p_i)$. 
Criticisms of the standard model

- Does not fit all market tranche prices on a given day.
- No dynamics, so no natural hedging strategy.
- Link to any observable correlation is tenuous. At best, model is “inspired” by Merton framework, so correlation is on equities.
Model flavors

Most numerical techniques rely on integrating over $Z$, given which all defaults are conditionally independent.

- Granular model—use full information on underlying spreads, and model full discrete loss distribution.
- Homogeneity assumption—assume all names in the portfolio have the same spread; use index level.
- Fine grained limit—continuous distribution, easy integrals
- Large pool model—combine homogeneity and fine grained assumptions.
- Also the question of whether to use full spread term structures or a single point
Correlation conventions

Start to introduce model abuses, ala the B-S volatility smile.

- **Compound correlations**
  - Price each individual tranche with a distinct correlation.
  - Not all tranches are monotonic in correlation.
  - Trouble calibrating mezzanine tranches, especially in 2005

- **Base correlations**
  - Decompose each tranche into “base” (i.e. 0-x%) tranches.
  - Bootstrap to calibrate all tranches.
  - Base tranches monotonic, but calibration not guaranteed.
Correlation conventions

- **Constant moneyness (ATM) correlations**
  - Some movements in implied correlation are due to changing “moneyness” as the index changes.
  - Examine correlations associated with a detachment point equal to implied index expected loss.
  - If base correlations are “sticky strike”, then ATM correlations are closer to “sticky delta”.

CDX correlation structure
CDX base correlations, granular model
CDX base correlations, large pool model
CDX base correlations, large pool model, with DJX option-implied correlation
CDX Series 4-7, GR model
CDX Series 4-7, LP model

Detachment plus index EL (%) vs. Base correlation (%)
CDX Series 8-9, GR model
CDX Series 8-9, LP model
Time series properties

- Examine the correlation between various implied correlations and the index.
- We would like this to be low. Why?
  - For risk, we have identified idiosyncratic risk correctly.
  - For hedging, we have captured what we are able to from the underlying.

- Start with statistics on daily changes.
CDX 0-3%, large pool model, base correlations

[Bar chart showing correlation to index (%) for Series 4, 5, 6, 7, 8, 9, and All for both Flat curve and Full curve.]
CDX 0-3%, base correlations
CDX 0-3%, large pool model
CDX 0-3%

Correlation to index (%)

LP, base
Gran, ATM

Series

4 5 6 7 8 9 All

-30
-20
-10
0
10
20
30
40
50
60

Risk Management
CDX 3-7%
Time series properties

Now look at statistics on weekly changes.
CDX 0-3%, large pool model, base correlations
CDX 0-3%, base correlations
CDX 0-3%, large pool model

Correlation to index (%)

Series

Base Corr
ATM Corr
CDX 0-3%
CDX 3-7%

![CDX 3-7% Chart](image)
Time series conclusions

- Looks like granular model produces “more idiosyncratic” implied correlations.
- The ATM approach looks appears to improve things, but seems to overcorrect in the most volatile periods.
- High correlations overall suggest index moves may have some predictive power for implied correlations.
- Differences are much more pronounced at a one-day horizon than a one-week horizon.
Hedging experiment

- On day zero, we have all prices (spreads and tranches) plus history.
- At a future date, we are told how the underlying spreads (index or single-name CDS) have moved, and are asked to guess what the new tranche prices should be.
- Compare the predicted tranche price moves to the actual ones, over time and across different modeling approaches.
- Another approach would be to look at delta hedge performance, but this mixes in the error from linearization.
Hedging experiment candidates

**Models**
- **Regression**—fit linear relationship between tranche price changes and index changes, using prior six months of data.
- **Large pool model**—calibrate based on current index levels.
- **Granular model**—calibrate based on current single-name CDS levels.

**Correlation approaches**
- **Base**—use most recent calibrated correlation.
- **ATM**—move along the most recent correlation structure according to change in the index-implied expected loss.
- **Regression**—fit linear relationship between base correlation changes and index changes, using prior six months of data.
CDX 0-3%, Regression
CDX 0-3%, LP model, base correlations
CDX 0-3%, Regression

![Graph showing the relationship between tranche price change and predicted change for different series. The graph includes data points for Series 4 to 9, with each series represented by a different marker color. The x-axis represents tranche price change (%), and the y-axis represents predicted change (%).](image-url)
CDX 0-3%, LP model, base correlations
Statistics on hedging approaches

- Examine standard deviation of tranche forecast error.
- Daily horizon

- Bars are for correlation approaches: Base (blue), ATM (green), Regression (red).
- Curve is for simple regression model.
CDX 0-3%, LP model
CDX 0-3%, GR model
CDX 3-7%, LP model
CDX 3-7%, GR model
CDX 7-10%, LP model
CDX 7-10%, GR model
Statistics on hedging approaches

- Examine correlation of tranche forecast error with actual tranche move.
- Daily horizon

- Bars are for correlation approaches: Base (blue), ATM (green), Regression (red).
- Curve is for simple regression model.
CDX 0-3%, LP model
CDX 0-3%, GR model
CDX 3-7%, LP model
CDX 3-7%, GR model
CDX 7-10%, LP model
CDX 7-10%, GR model
Statistics on hedging approaches

- Examine standard deviation of tranche forecast error.
- Weekly horizon

- Bars are for correlation approaches: Base (blue), ATM (green), Regression (red).
- Curve is for simple regression model.
CDX 0-3%, LP model

The image shows a bar chart with the x-axis labeled as "Series" and the y-axis labeled as "STD of prediction error". The chart compares the standard deviation of prediction error across different series, ranging from 4 to 9, and an 'All' category. Each series is represented by bars in different colors, with additional line plots for some series.
CDX 0-3%, GR model

STD of prediction error

Series: 4 5 6 7 8 9 All
CDX 3-7%, LP model
CDX 3-7%, GR model
CDX 7-10%, LP model
CDX 7-10%, GR model
Statistics on hedging approaches

- Examine correlation of tranche forecast error with actual tranche move.
- Weekly horizon

- Bars are for correlation approaches: Base (blue), ATM (green), Regression (red).
- Curve is for simple regression model.
CDX 0-3%, LP model
CDX 0-3%, GR model
CDX 3-7%, LP model
CDX 3-7%, GR model

![Graph showing correlation between actual and error for different series. The x-axis represents series numbers 4 to 9 and 'All', while the y-axis shows correlation values ranging from -0.9 to 0.1. The graph illustrates the correlation for each series and the overall 'All' series.]
CDX 7-10%, LP model
CDX 7-10%, GR model
Conclusions

- Overall, hedging errors are surprisingly large.
- For the equity tranche, a simple regression works quite well, with its underhedging problems reduced at a slightly longer horizon.
- There is a benefit to using the granular model, but is it worth the cost?
- For more senior tranches, the ATM approach appears to capture some of the link to credit spreads, but does not markedly reduce the hedging error.

- Any candidate for a new standard model should be able to do better in this experiment.
Further investigations

- Can we use these results to learn more about how our model might be misspecified?
  - Richer correlation structure?
  - Different copula?
  - Better dynamics?

- How much worse (or better) are compound correlations?
- Can the regression be improved by including the HiVol index?
- Can the LP model be improved by adding a simple correction for heterogeneity?
- Does any of this change at or close to defaults?
Further reading


All are available at www.riskmetrics.com