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Credit Index Options: the no-armageddon pricing measure and the role of correlation after the subprime crisis

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Outline

- The Forward Credit Index and Credit Index Options
- The flaws in the market approach
- Pricing without Armageddon
- A consistent Index Spread
- Arbitrage-free pricing of Credit Options
- The role of correlation
- Credit Crisis and Mispricing between Market and No-Arbitrage Formulas

The Credit Index

Portfolio of n names with initial notional $N(0) = 1$. Each name has notional $\frac{1}{n}$. τ_i default time of name i, with associated loss $\left(1 - Rec^i\right)\frac{1}{n}$. Total portfolio loss

$$L(t) = \frac{1}{n}\sum_{i=1}^{n} 1_{\{\tau_i < t\}}\left(1 - Rec^i\right) = L(t) = \frac{1}{n}(1 - R)\sum_{i=1}^{n} 1_{\{\tau_i < t\}}$$

where the second equality holds for a flat recovery rate R. At time t, the Outstanding Notional is $N_t = 1 - L(t)/(1 - R)$.

The **Protection Leg** pays, at default of one name in the index, the corresponding loss. At τ_i : $dL(\tau_i) = (1 - R)\,1/n$
from start date T_A to T_M or until all names default.

The Index payments

The **Premium Leg**, at payment times T_j, $j = A+1, \ldots, M$ (or until all names have defaulted), with year fractions α_j, pays a premium K on the daily average $\hat{N}(T_j)$ of the outstanding notional $N(t)$ for $t \in (T_{j-1}, T_j]$.

$$\text{At } T_j : \qquad \hat{N}(T_j) \times \alpha_j \times K$$

The discounted payoff of the Protection Leg is

$$Prot_t^{T_A,T_M} = \int_{T_A}^{T_M} D(t,u)\, dL(u) \approx \sum_{j=A+1}^{M} D(t,T_j) \left[L(T_j) - L(T_{j-1}) \right]$$

By $D(t,T)$ we indicate the discount factor from T to t. Its expectation is the corresponding bond price, $P(t,T) = \mathbb{E}^Q \left[D(t,T)\mathbf{1} | \mathcal{F}_t \right]$.

The Index payments

The discounted payoff of the Premium Leg is

$$Prem_t^{T_A,T_M}(K) = \left\{ \sum_{j=A+1}^{M} D(t,T_j) \int_{T_{j-1}}^{T_j} N(t)\,dt \right\} \times K \approx$$

$$\left\{ \sum_{j=A+1}^{M} D(t,T_j)\,\alpha_j N(T_j) \right\} K = \left\{ \sum_{j=A+1}^{M} D(t,T_j)\,\alpha_j \left(1 - \frac{L(T_j)}{(1-R)}\right) \right\} K$$

The quantity in curly brackets is called $\gamma_t^{T_A,T_M}$.

Evaluating Premium and Protection Legs

The value of the two legs is computed by expectation under the risk neutral probability measure $\mathbb{Q}$. We use the notation $\Pi(X_t, s) = \mathbb{E}[X_t | \mathcal{F}_s]$. When $s = t$, we omit the argument s writing simply $\Pi(X_t)$. The Payer Forward Index starting at T_A and lasting until T_M has a price

$$\Pi\left(I_t^{T_A,T_M}(K)\right) := \mathbb{E}\left[I_t^{T_A,T_M}(K)\middle| \mathcal{F}_t\right] := \mathbb{E}\left[Prot_t^{T_A,T_M} - Prem_t^{T_A,T_M}(K)\middle| \mathcal{F}_t\right]$$

Substituting the payoffs, one sees that the Index value does not depend on the Loss distribution but only on the Expected Loss $\boxed{\mathbb{E}_t[L(T)]}$ at different maturities.

The Index Spread

In the simplest definition, the equilibrium spread at time t is the value of the spread K that sets the value of the forward index to zero at time t:

$$S_t^{T_A,T_M} = \frac{\Pi\left(Prot_t^{T_A,T_M}\right)}{\Pi\left(\gamma_t^{T_A,T_M}\right)}.$$

allowing to write the index value as

$$\Pi\left(I_t^{T_A,T_M}(K)\right) = \Pi\left(\gamma_t^{T_A,T_M}\right)\left(S_t^{T_A,T_M} - K\right)$$

Index Options

A **payer** Index Option with inception 0, strike K and exercise T_A, written on an index with maturity T_M, gives the right but no obligation to *enter at T_A into the running Index with final payment at T_M* **as protection buyer** *paying a fixed rate K*.

However, the option buyer would give away protection from inception 0 to maturity T_A. In order to attract investors, standard Credit Index Option payoff includes the payment of the losses from the option inception to T_A as well (**front end protection**)

$$F_t^{T_A} = D(t, T_A) L(T_A) \quad \Pi\left(F_t^{T_A}\right) = \mathbb{E}\left[D(t, T_A) L(T_A) | \mathcal{F}_t\right].$$

The rough approach

In the roughest approach, the option payoff in case of exercise

$$\Pi\left(I_{T_A}^{T_A,T_M}(K)\right)+F_{T_A}^{T_A}=\Pi\left(\gamma_{T_A}^{T_A,T_M}\right)\left(S_{T_A}^{T_A,T_M}-K\right)+F_{T_A}^{T_A}$$

is improperly evaluated as ($(\cdot)^+$ should include all parts)

$$\mathbb{E}\left[D\left(t,T_A\right)\Pi\left(\gamma_{T_A}^{T_A,T_M}\right)\left(S_{T_A}^{T_A,T_M}-K\right)^+\middle|\mathcal{F}_t\right]+\mathbb{E}\left[F_t^{T_A}\middle|\mathcal{F}_t\right]$$

and then expressing the 1st comp. through a std Black formula

$$\Pi\left(\gamma_t^{T_A,T_M}\right)Black\left(S_t^{T_A,T_M},K,\sigma^{T_A,T_M}\sqrt{T_A-t}\right)+\Pi\left(F_t^{T_A}\right),$$

Pedersen (2003) properly includes the front-end protection into the option and through numerical integration gets to an improved Black formula.

The market approach

$$\tilde{I}_t^{T_A,T_M}(K) = Prot_t^{T_A,T_M} - Prem_t^{T_A,T_M}(K) + F_t^{T_A}.$$

It is natural to give a new spread definition, setting to zero $\Pi\left(\tilde{I}_t^{T_A,T_M}(K)\right)$.
This leads to the *Loss-Adjusted Market Index Spread*

$$\tilde{S}_t^{T_A,T_M} = \left[\Pi\left(Prot_t^{T_A,T_M}\right) + \Pi\left(F_t^{T_A}\right)\right] / \Pi\left(\gamma_t^{T_A,T_M}\right)$$

that allows to write the option price as

$$\mathbb{E}\left[D(t,T_A)\left(\Pi\left(\gamma_{T_A}^{T_A,T_M}\right)\left(\tilde{S}_{T_A}^{T_A,T_M} - K\right)\right)^+ \middle| \mathcal{F}_t\right].$$

"Numeraire" $\Pi\left(\gamma_{T_A}^{T_A,T_M}\right)$ and lognormal $\tilde{S}_t^{T_A,T_M}$: Mkt Index Option Form.:

$$\Pi\left(\gamma_t^{T_A,T_M}\right) Black\left(\tilde{S}_t^{T_A,T_M}, K, \tilde{\sigma}^{T_A,T_M}\sqrt{T_A}\right). \quad (1)$$

The three flaws of the Standard Formula

1. The definition of $\tilde{S}_t^{T_A,T_M}$ is valid only when the denominator

$$\Pi\left(\gamma_t^{T_A,T_M}\right) = \sum_{j=A+1}^{M} \mathbb{E}\left[D\left(t,T_j\right)\alpha_j\left(1-\frac{L\left(T_j\right)}{(1-R)}\right)\middle|\mathcal{F}_t\right]$$

   is different from zero. Since $\Pi\left(\gamma_t^{T_A,T_M}\right)$ is the price of a portfolio of defaultable assets, it can go to zero.

2. When $\Pi\left(\gamma_t^{T_A,T_M}\right)=0$ the pricing formula (1) is undefined, and in this scenario $\tilde{S}_t^{T_A,T_M}$ does not set the value of the adjusted index to zero.

3. If we worked conditionally on $\Pi\left(\gamma_t^{T_A,T_M}\right)>0$, the resulting "survival measure" would not be equivalent to the standard risk-neutral measure.

Subfiltration Pricing without armageddon

We adapt a technique used earlier by Jamshidian (2004) and Brigo (2005) for single name CDS options, making use of a *subfiltration structure*, separating default free information from information on the default event. This is based on the Jeanblanc-Rutkowski (J-R) filtration-switching formula.

To effectively use J-R in index options context, define

$$\hat{\tau} = \max\left(\tau_1, \tau_2, \ldots, \tau_n\right)$$

and define a new filtration $\hat{\mathcal{H}}_t$ such that

$$\mathcal{F}_t = \hat{\mathcal{J}}_t \vee \hat{\mathcal{H}}_t \ , \quad \hat{\mathcal{J}}_t = \sigma\left(\{\hat{\tau} > u\}, u \leq t\right),$$

so that $\hat{\mathcal{H}}_t$ excludes information on the portfolio "armageddon event".

A DV01 γ without Armageddon

Define

$$\hat{\Pi}\left(\gamma_t^{T_A,T_M}\right) := \mathbb{E}\left[\gamma_t^{T_A,T_M}|\hat{\mathcal{H}}_t\right].$$

Exploiting $\gamma_t^{T_A,T_M} = \mathbf{1}_{\{\hat{\tau}>t\}}\gamma_t^{T_A,T_M}$, which is necessary to be able to apply the J-R-type formula wrt the filtration $\mathcal{H}_t$

$$\begin{aligned}\Pi\left(\gamma_t^{T_A,T_M}\right) &= \mathbb{E}\left[\gamma_t^{T_A,T_M}|\mathcal{F}_t\right] = \frac{\mathbf{1}_{\{\hat{\tau}>t\}}}{\mathbb{Q}\left(\hat{\tau} > t|\hat{\mathcal{H}}_t\right)}\mathbb{E}\left[\gamma_t^{T_A,T_M}|\hat{\mathcal{H}}_t\right] \\ &= \frac{\mathbf{1}_{\{\hat{\tau}>t\}}}{\mathbb{Q}\left(\hat{\tau} > t|\hat{\mathcal{H}}_t\right)}\hat{\Pi}\left(\gamma_t^{T_A,T_M}\right). \qquad (2)\end{aligned}$$

The quantity $\hat{\Pi}\left(\gamma_t^{T_A,T_M}\right)$ is never null, and we will see that it is what we need for an effective definition of the Index Spread and of an equivalent pricing measure for Index Options.

The Arbitrage-free Index Spread

Applying a J-R type formula to the loss adjusted index does not work, since the necessary condition does not hold:

$$\mathbf{1}_{\{\hat{\tau}>t\}} \tilde{I}_t^{T_A,T_M} \neq \tilde{I}_t^{T_A,T_M}$$

Indeed, the Loss-Adjusted index, differently from $\gamma_t^{T_A,T_M}$, is never null (even in case $\hat{\tau} \leq t$ we receive front end protection). Write then

$$\begin{aligned}
\mathbb{E}\left[\mathbf{Y}_t^T | \mathcal{F}_t\right] &= \mathbb{E}\left[\mathbf{1}_{\{\hat{\tau}\leq t\}} \mathbf{Y}_t^T | \mathcal{F}_t\right] + \mathbb{E}\left[\mathbf{1}_{\{\hat{\tau}>t\}} \mathbf{Y}_t^T | \mathcal{F}_t\right] \\
&= \mathbf{1}_{\{\hat{\tau}\leq t\}} \mathbb{E}\left[\mathbf{Y}_t^T | \sigma\left(\hat{\tau}\right) \vee \hat{\mathcal{H}}_t\right] + \frac{\mathbf{1}_{\{\hat{\tau}>t\}}}{\mathbb{Q}\left(\hat{\tau} > t | \hat{\mathcal{H}}_t\right)} \mathbb{E}\left[\mathbf{1}_{\{\hat{\tau}>t\}} \mathbf{Y}_t^T | \hat{\mathcal{H}}_t\right]
\end{aligned}$$

The 1st component corresponds to the value when we know that all defaults have already happened, and we know the exact "armageddon" time.

The Arbitrage-free Index Spread

$$\Pi\left(\tilde{I}_t^{T_A,T_M}(K)\right) = \Pi(Prot_t^{T_A,T_M} - Prem_t^{T_A,T_M}(K) + F_t^{T_A}) \ = \tag{3}$$

$$= \quad \frac{\mathbf{1}_{\{\hat{\tau}>t\}}}{\mathbb{Q}\left(\hat{\tau} > t|\hat{\mathcal{H}}_t\right)}\left\{\hat{\Pi}\left(Prot_t^{T_A,T_M}\right) - \hat{\Pi}\left(Prem_t^{T_A,T_M}(K)\right) + \mathbb{E}\left[\mathbf{1}_{\{\hat{\tau}>T_A\}}F_t^{T_A}|\hat{\mathcal{H}}_t\right]\right\}$$

$$+\frac{\mathbf{1}_{\{\hat{\tau}>t\}}(1-R)}{\mathbb{Q}\left(\hat{\tau} > t|\hat{\mathcal{H}}_t\right)}\times\mathbb{E}\left[\mathbf{1}_{\{t<\hat{\tau}\le T_A\}}D(t,T_A)|\hat{\mathcal{H}}_t\right] +$$

$$+\mathbf{1}_{\{\hat{\tau}\le t\}}(1-R)P(t,T_A)$$

This shows the actual **components of Loss-Adjusted index**, and will lead us to a consistent valuation of the Index Option. In a Loss-Adjusted portfolio we cannot define in all scenarios the equilibrium spread as the value of the spread zeroing the index value, as there are always scenarios where the index is non zero, regardless of the spread.

The Arbitrage-free Index Spread

The financially meaningful definition of the Index Spread considers the level of K setting the Index value to 0 in all scenarios where some names survive until maturity. Only in such scenarios the payoff actually depends on K. Set to 0 only the first component of the index value (3),

$$\frac{\mathbf{1}_{\{\hat{\tau}>t\}}}{\mathbb{Q}\left(\hat{\tau}>t|\hat{\mathcal{H}}_t\right)}\left\{\hat{\Pi}\left(Prot_t^{T_A,T_M}\right)-\hat{\Pi}\left(Prem_t^{T_A,T_M}(K)\right)+\mathbb{E}\left[\mathbf{1}_{\{\hat{\tau}>T_A\}}F_t^{T_A}|\hat{\mathcal{H}}_t\right]\right\},$$

which is the price of an armageddon-knock out tradable asset. We obtain the following definition of the equilibrium *Arbitrage-free Index Spread*

$$\hat{S}_t^{T_A,T_M}=\left[\hat{\Pi}\left(Prot_t^{T_A,T_M}\right)+\mathbb{E}\left[\mathbf{1}_{\{\hat{\tau}>T_A\}}F_t^{T_A}|\hat{\mathcal{H}}_t\right]\right]/\hat{\Pi}\left(\gamma_t^{T_A,T_M}\right) \tag{4}$$

This definition of the index spread is both regular, since $\hat{\Pi}\left(\gamma_t^{T_A,T_M}\right)$ is bounded away from zero, and has a reasonable financial meaning.

The Option Formula Components

$$\Pi\left(Option_t^{T_A,T_M}(K)\right) = \mathbb{E}\left[D(t,T_A)\left(\Pi\left(\tilde{I}_{T_A}^{T_A,T_M}(K)\right)\right)^+ |\mathcal{F}_t\right] = \tag{5}$$

$$\frac{\mathbf{1}_{\{\hat{\tau}>t\}}}{\mathbb{Q}\left(\hat{\tau}>t|\hat{\mathcal{H}}_t\right)}\mathbb{E}\left[D(t,T_A)\frac{\mathbf{1}_{\{\hat{\tau}>T_A\}}}{\mathbb{Q}\left(\hat{\tau}>T_A|\hat{\mathcal{H}}_{T_A}\right)}\hat{\Pi}\left(\gamma_{T_A}^{T_A,T_M}\right)\left(\hat{S}_{T_A}^{T_A,T_M}-K\right)^+ |\hat{\mathcal{H}}_t\right]$$

$$+\frac{\mathbf{1}_{\{\hat{\tau}>t\}}}{\mathbb{Q}\left(\hat{\tau}>t|\hat{\mathcal{H}}_t\right)}\mathbb{E}\left[D(t,T_A)\mathbf{1}_{\{t<\hat{\tau}\leq T_A\}}(1-R)|\hat{\mathcal{H}}_t\right]$$

$$+\mathbf{1}_{\{\hat{\tau}\leq t\}}(1-R)P(t,T_A) =: \hat{O}_1+\hat{O}_2+\hat{O}_3$$

follows easily. $\hat{O}_3$ is the option value when "armageddon" is before t. $\hat{O}_2$ takes into account the probability of such an event between now and the option expiry. For $\hat{O}_1$, that was the only one considered in the simpler formula (1), we develop a standard option formula.

The numeraire

For that we need a change of measure, solving also Problem 3. This is technically demanding, but the preceding analysis helps.

$$\hat{O}_1 = \frac{\mathbf{1}_{\{\hat{\tau}>t\}}}{\mathbb{Q}\left(\hat{\tau} > t|\hat{\mathcal{H}}_t\right)}\mathbb{E}\left[D\left(t,T_A\right)\hat{\Pi}\left(\gamma_{T_A}^{T_A,T_M}\right)\left(\hat{S}_{T_A}^{T_A,T_M} - K\right)^+ |\hat{\mathcal{H}}_t\right]$$

Now it is natural to take the quantity

$$\hat{\Pi}\left(\gamma_t^{T_A,T_M}\right) = \mathbb{E}\left[\gamma_t^{T_A,T_M}|\hat{\mathcal{H}}_t\right] = \mathbb{E}\left[\sum_{j=A+1}^{M} D\left(t,T_j\right)\alpha_j\left(1 - \frac{L\left(T_j\right)}{\left(1-R\right)}\right)|\hat{\mathcal{H}}_t\right]$$

to define our probability measure $\hat{\mathbb{Q}}^{T_A,T_M}$. Differently from $\Pi\left(\gamma_t^{T_A,T_M}\right)$, the sub-filtration based $\hat{\Pi}\left(\gamma_t^{T_A,T_M}\right)$ is strictly positive.

The No-Armageddon Pricing Measure

We define the T_A, T_M*-no-armageddon pricing measure* $\hat{\mathbb{Q}}^{T_A,T_M}$ through definition of the Radon-Nykodim derivative with respect to $\mathbb{Q}$

$$Z_{T_A} = \left.\frac{d\hat{\mathbb{Q}}^{T_A,T_M}}{d\mathbb{Q}}\right|_{\hat{\mathcal{H}}_{T_A}} = \frac{B_0\ \hat{\Pi}\left(\gamma_{T_A}^{T_A,T_M}\right)}{\hat{\Pi}\left(\gamma_0^{T_A,T_M}\right)\ B_{T_A}}.$$

and we can compute

$$Z_t = \mathbb{E}\left[Z_{T_A}\middle|\hat{\mathcal{H}}_t\right] = \mathbb{E}\left[\left.\frac{d\hat{\mathbb{Q}}^{T_A,T_M}}{d\mathbb{Q}}\right|_{\hat{\mathcal{H}}_{T_A}}\middle|\hat{\mathcal{H}}_t\right] = \frac{B_0\hat{\Pi}\left(\gamma_t^{T_A,T_M}\right)}{\hat{\Pi}\left(\gamma_0^{T_A,T_M}\right)B_t}$$

Thus also the Radon-Nykodim derivative restricted to all $\hat{\mathcal{H}}_t$, $t \leq T_A$, can be expressed in closed form through market quantities. This is sufficient to apply the Bayes rule for conditional change of measure.

Bayes rule for conditional change of measure: Consider a sub σ-algebra $\mathcal{N}$ of σ-algebra $\mathcal{M}$ and an $\mathcal{M}$-measurable X, integrable under the measures $P1$ and $P2$, $P1 \sim P2$. Then

$$\mathbb{E}^{P1}\left[X \frac{\mathbb{E}^{P1}\left[\frac{dP2}{dP1}|\mathcal{M}\right]}{\mathbb{E}^{P1}\left[\frac{dP2}{dP1}|\mathcal{N}\right]} \middle| \mathcal{N} \right] = \mathbb{E}^{P2}\left[X|\mathcal{N}\right].$$

$$\begin{aligned}
\hat{O}_1 &= \frac{\mathbf{1}_{\{\hat{\tau}>t\}}}{\mathbb{Q}\left(\hat{\tau} > t|\hat{\mathcal{H}}_t\right)} \hat{\Pi}\left(\gamma_t^{T_A,T_M}\right) \mathbb{E}\left[\frac{\mathbb{E}\left[\frac{d\hat{\mathbb{Q}}^{T_A,T_M}}{d\mathbb{Q}}|\hat{\mathcal{H}}_{T_A}\right]}{\mathbb{E}\left[\frac{d\hat{\mathbb{Q}}^{T_A,T_M}}{d\mathbb{Q}}|\hat{\mathcal{H}}_t\right]} \left(\hat{S}_{T_A}^{T_A,T_M} - K\right)^+ |\hat{\mathcal{H}}_t \right] \\
&= \frac{\mathbf{1}_{\{\hat{\tau}>t\}}}{\mathbb{Q}\left(\hat{\tau} > t|\hat{\mathcal{H}}_t\right)} \hat{\Pi}\left(\gamma_t^{T_A,T_M}\right) \hat{\mathbb{E}}^{T_A,T_M}\left[\left(\hat{S}_{T_A}^{T_A,T_M} - K\right)^+ |\hat{\mathcal{H}}_t\right] \\
&= \Pi\left(\gamma_t^{T_A,T_M}\right) \hat{\mathbb{E}}^{T_A,T_M}\left[\left(\hat{S}_{T_A}^{T_A,T_M} - K\right)^+ |\hat{\mathcal{H}}_t\right]
\end{aligned}$$

Arbitrage-free Credit Index Option Formula

$$\hat{O}_1 = \Pi\left(\gamma_t^{T_A,T_M}\right)\hat{\mathbb{E}}^{T_A,T_M}\left[\left(\hat{S}_{T_A}^{T_A,T_M} - K\right)^+ |\hat{\mathcal{H}}_t\right]$$

We have also that $\hat{S}_t^{T_A,T_M}$ is a $\hat{\mathcal{H}}_t$-martingale under $\hat{\mathbb{Q}}^{T_A,T_M}$. Assuming lognormality (*not necessary, can use any smile martingale dynamics*) $d\hat{S}_t^{T_A,T_M} = \hat{\sigma}^{T_A,T_M}\hat{S}_t^{T_A,T_M} dV^{T_A,T_M}, \quad t \leq T_a$ we have the following *Arbitrage-free Credit Index Option formula*

$$\Pi\left(Option_t^{T_A,T_M}(K)\right) = \Pi\left(\gamma_t^{T_A,T_M}\right) Black\left(\hat{S}_0^{T_A,T_M}, K, \hat{\sigma}^{T_A,T_M}\sqrt{T_A}\right) \tag{6}$$

$$+\frac{\mathbf{1}_{\{\hat{\tau}>t\}}}{\mathbb{Q}\left(\hat{\tau}>t|\hat{\mathcal{H}}_t\right)}\mathbb{E}\left[D(t,T_A)\,\mathbf{1}_{\{t<\hat{\tau}\leq T_A\}}(1-R)\,|\hat{\mathcal{H}}_t\right]$$

$$+\mathbf{1}_{\{\hat{\tau}\leq t\}}(1-R)\,P(t,T_A)$$

Implementing the Arbitrage-free Formula

How is the formula practically implemented?

$$\Pi\left(Option_0^{T_A,T_M}(K)\right) =$$

$$\Pi\left(\gamma_0^{T_A,T_M}\right) Black\left(\hat{S}_0^{T_A,T_M}, K, \hat{\sigma}^{T_A,T_M}\sqrt{T_A}\right) + \boxed{(1-R)\,P(0,T_A)\,\mathbb{Q}(\hat{\tau} \leq T_A)}$$

$$= \Pi\left(\gamma_0^{T_A,T_M}\right)\left[Bl\left(\tilde{S}_0^{T_A,T_M} - \frac{\boxed{(1-R)\,P(0,T_A)\,\mathbb{Q}(\hat{\tau} \leq T_A)}}{\Pi\left(\gamma_0^{T_A,T_M}\right)}, K, \hat{\sigma}^{T_A,T_M}\sqrt{T_A}\right)\right.$$

$$\left. + \boxed{(1-R)\,P(0,T_A)\,\mathbb{Q}(\hat{\tau} \leq T_A)}\right.$$

Now we apply the formula in practice.

Credit Index Options before and after 2007 subprime crisis

For computing $\hat{\tau}$ probabilities, staying close to correlation coming from liquid CDO tranches on the same pool, one needs the correlations associated to the most senior tranche $\left[\frac{n-1}{n}(1-R),(1-R)\right]$ (very thin), at a short maturity.

Market agrees correlation increases with seniority and decreases with maturity. We expect a correlation higher than the highest level quoted by the CDX market, $\rho_{30\%}$.

However we consider a range of equally spaced correlations in-between i-Traxx and CDX most seniors, as this choice tends to *underestimate* the probability of $\hat{\tau}$, compared to the standard market approach of extrapolating correlations.

Thus the relevance of the new formula will be underestimated, and if we find a relevant impact this implies the actual impact will be even larger.

Options on i-Traxx Europe Main - 2007 vs 2008

In the next table we report the market inputs. The bid-offer spread for options in March 08 was in the range 5-8 bps.

	March-09-07	March-11-08
Spot Spread 5y: $S_0^{9m,5y}$	22.50 bp	154.50 bp
Forward Spread Adjusted 9m-5y: $\tilde{S}_0^{9m,5y}$	23.67 bp	163.60 bp
Implied Volatility, $K = \tilde{S}_0^{9m,5y} \times 0.9$	52%	108%
Implied Volatility, $K = \tilde{S}_0^{9m,5y} \times 1.1$	54%	113%
Correlation 22% I-Traxx Main: $\rho_{0.22}^I$	0.545	0.912
Correlation 30% CDX IG: $\rho_{0.3}^C$	0.701	0.999
Annuity 9m-5y: $\Pi\left(\gamma_0^{9m,5y}\right)$	3.993	3.912

Market Inputs: : March-09-07 (left), March-11-08 (right)

Options on i-Traxx Europe Main - March 2007

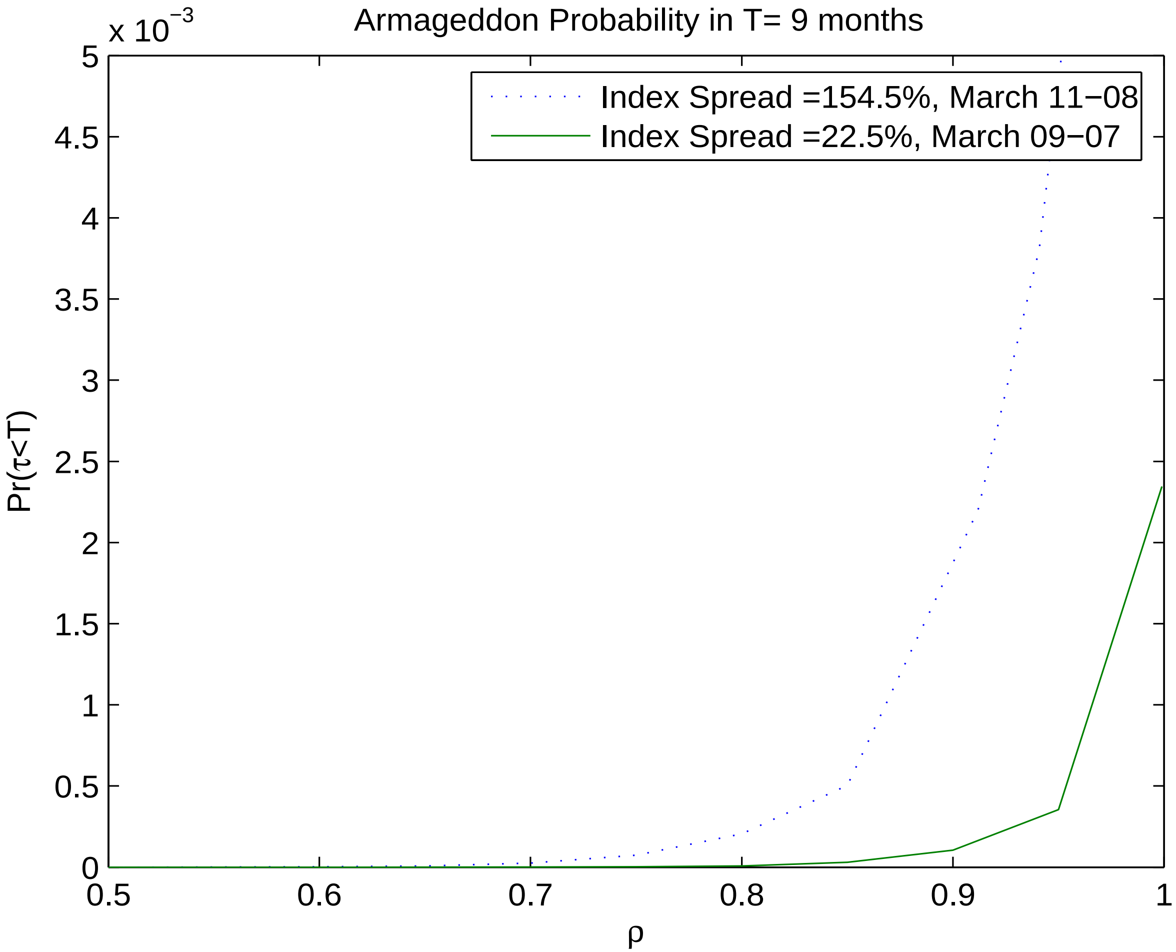
Strike (Call)	26	21
Market Formula	**23.289**	**11.619**
No-Arb. Form. $\rho = 0.545$	23.289	11.619
No-Arb. Form. $\rho = 0.597$	23.289	11.619
No-Arb. Form. $\rho = 0.649$	23.289	11.618
No-Arb. Form. $\rho = 0.701$	23.286	11.614
Strike (Put)	26	21
Market Formula	**13.840**	**21.069**
No-Arb. Form. $\rho = 0.545$	13.840	21.069
No-Arb. Form. $\rho = 0.597$	13.840	21.069
No-Arb. Form. $\rho = 0.649$	13.840	21.069
No-Arb. Form. $\rho = 0.701$	13.843	21.071

March-09-07 Options on i-Traxx 5y, Maturity 9m

Options on i-Traxx Europe Main - March 2008

Strike (Call)	180	147
Market Formula	**286.241**	**189.076**
No-Arb. Form. $\rho = 0.912$	277.668	179.624
Difference	**8.573**	**9.453**
No-Arb. Form. $\rho = 0.941$	271.460	172.769
Difference	**14.781**	**16.307**
No-Arb. Form. $\rho = 0.970$	258.887	158.862
Difference	**27.354**	**30.215**
No-Arb. Form. $\rho = 0.999$	212.867	107.630
Difference	**73.374**	**81.447**

March-11-08 Options on i-Traxx 5y, Maturity 9m


Armageddon Probability in T= 9 months
x 10^-3
Index Spread =154.5%, March 11−08
Index Spread =22.5%, March 09−07
Pr(τ<T)
ρ
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
0.5
0.6
0.7
0.8
0.9
1

Figure