**Integrality Theorem**

Assuming integer data, every basic feasible solution assigns integer flow to every arc.

**Corollary**

Assuming integer data, every basic optimal solution assigns integer flow to every arc.


\textbf{Transportation Problem}

Each node is one of two types:

- source (supply) node
- destination (demand) node

Every arc has:

- its tail at a supply node
- its head at a demand node

Such a graph is called \textbf{bipartite}.
Solving with Pivot Tool

Best to arrange:

- supply nodes vertically on left
- demand nodes horizontally across top

Data:

Note that arc data appears as a neat table.
Tree Solution

Leaving arc: (a,b)
Entering arc: (i,h)

Etc., etc., etc.
Assignment Problem

Transportation problem in which

- Equal number of supply and demand nodes.
- Every supply node has a supply of one.
- Every demand node has a demand for one.
- Each supply node is connected to every demand node (called a complete bipartite graph).
- Solution is required to be all integers.

Notes:

- These problems are very common.
- They are notoriously degenerate ($2n$ constraints but only $n$ nonzero flows).