

COMMERCIAL AUTO INSURANCE
RISK MANAGEMENT STRATEGIES

THOMAS P. BYRNE

ADVISOR: PROFESSOR ALAIN L. KORNHAUSER

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In loving memory of my cousin, Matthew.

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Stellis Aequus Durando,

Tom

Abstract

Autopilot, Distronic Plus, Intelligent Cruise Control. These are the names of semi-autonomous driving systems currently offered by Tesla, Mercedes-Benz, and Infiniti, respectively. It is clear that the leading companies in the automotive industry have embraced the push for autonomous vehicle technologies, and that this trend will continue to reshape our fundamental notions of transportation and mobility in the coming years. Until autonomous vehicles become a part of our daily lives, businesses in industries related to the transportation sector will be forced to evaluate their present business operations, and to decide if embracing autonomous vehicle technologies is a valuable and worthwhile investment with respect to the profitability of their respective businesses.

This thesis analyzed the performance of a specialty commercial auto insurance company's currently available risk management strategies, as well as the potential effectiveness and profitability of autonomous vehicle technologies if purposed as risk management strategies. The purpose of this thesis was to conduct a case study which rigorously analyzed the current risk management practices of a commercial auto insurance company, to propose recommendations for the company in the case study to improve their risk management strategies, and to investigate the potential of new risk management strategies which utilize autonomous vehicle technologies.

Four existing risk management strategies were analyzed in this thesis: the use of automated event recorders (abbr. AERs), defensive driving courses and training, physical abilities testing (abbr. PAT) programs, and return-to-work (abbr. RTW) programs. After a comprehensive analysis, it was determined that insured customers from the trucking industry who had implemented PAT programs correlated with outperforming the baseline claims frequency standard by at least 0.09 claims per one million miles driven, and this figure is confirmed at the 95% confidence level. Ultimately, the existing risk management strategies analyzed in this thesis were not supported by the data to have a consistent correlation with outperforming the baseline performance standards. It is recommended that risk management strategies involving autonomous vehicle technologies be investigated by the Company.

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1 Introduction

The primary source of data for this thesis is a specialty commercial auto insurance company whose book of business spans multiple transportation sub-industries (e.g. passenger transportation, trucking, crane & rigging, etc.) and includes insured customers located throughout the United States. The company providing the data for this thesis, to be referred to simply as “the Company” hereafter, has permitted the use of anonymized data regarding the commercial auto insurance claims history of and risk management services provided to a subset of its insured customers.

This thesis examines the Company as a case study with regard to the performance of current risk management strategies as implemented by the insured customers of a commercial auto insurance company. The four risk management strategies analyzed in this thesis include:

- Automated event recorders (abbr. AERs)
- Defensive driving courses and training
- Physical abilities testing (abbr. PAT) programs
- Return-to-work (abbr. RTW) programs

The data provided by the Company comprises three hundred twenty-five (325) unique customers. Implementation of the four risk management strategies by the number of customers from the data is included in Table 1 below:

Table 1: Insured Customers by Risk Management Strategy Participation

Strategy	Number of Insured Customers	Percentage
AERs	222	68.30%
Defensive Driving Courses	74	22.80%
PAT Programs	36	11.10%
RTW Programs	66	20.30%

From the subset of insured customers in the data provided by the Company, the use of automated event recorders ranks as the most frequently implemented risk management strategy by far, with 68% of sample customers using AERs (222 customers). This participation rate is at least triple the number

of customers whose risk management strategies include defensive driving courses (74 customers), physical abilities testing programs (36 customers), or return-to-work programs (66 customers).

The use of these four risk management strategies is abundant among the sample of insured customers included in the Company's data, with 77% of customers in the data reported as using at least one of the strategies. Given that a robust statistical analysis of these risk management strategies' empirical effectiveness has not previously been performed with the Company's data, the motivation for this thesis thus arises. Questions that this thesis addresses include:

- Is an insured customer's use of any of the currently available risk management strategies correlated with significantly better performance with regard to the frequency and severity of commercial auto claims?
- What recommendations can be made to the Company with regard to currently available risk management strategies to improve the performance of its insured customers?
- If the currently available risk management strategies prove to be ineffective, what additional products and services have the potential to be purposed as risk management strategies?

Two products and services that may have the potential to be purposed as risk management strategies are introduced in the literature review contained in Section 2. The remainder of this section provides an overview of the four aforementioned risk management strategies.

1.1 Automated Event Recorders (abbr. AERs)

The most widely implemented risk management strategy analyzed in this thesis is the use of automated event recorders (abbr. AERs), which comprises 64% of trucking customers (83 of 129) and 71% of passenger transportation companies (139 of 196) from the sample data. An AER is a video camera equipped with an accelerometer that captures high intensity events such as collisions, hard braking, and swerving. The purpose of installing AERs in a fleet of commercial vehicles is to identify and subsequently correct unsafe driving habits observed when an event is triggered by the AER's accelerometer.

As of July 2016, the Company’s insured customers have accounted for the installation of more than twenty-eight thousand AERs in fleets across the country. A risk management firm (independent from the Company) often offers a subsidy program in which it agrees to pay for a portion of the AER hardware cost in exchange for insured customers agreeing to meet specific key performance indicators (abbr. KPIs) regarding the quality of their implementation of the AERs.

This thesis analyzes the effectiveness of AER use as a risk management strategy considering variables such as a customer’s fleet size, and whether a customer’s AER implementation was subjectively judged as best-in-class or worst-in-class by interviewed risk management specialists. These results are contained in Subsection 5.1.

To provide some context as to the extent of AERs as a risk management strategy and the growth rate associated with AER use amongst the Company’s insured customers, a brief analysis of AER purchase orders by the independent risk management firm is included below.

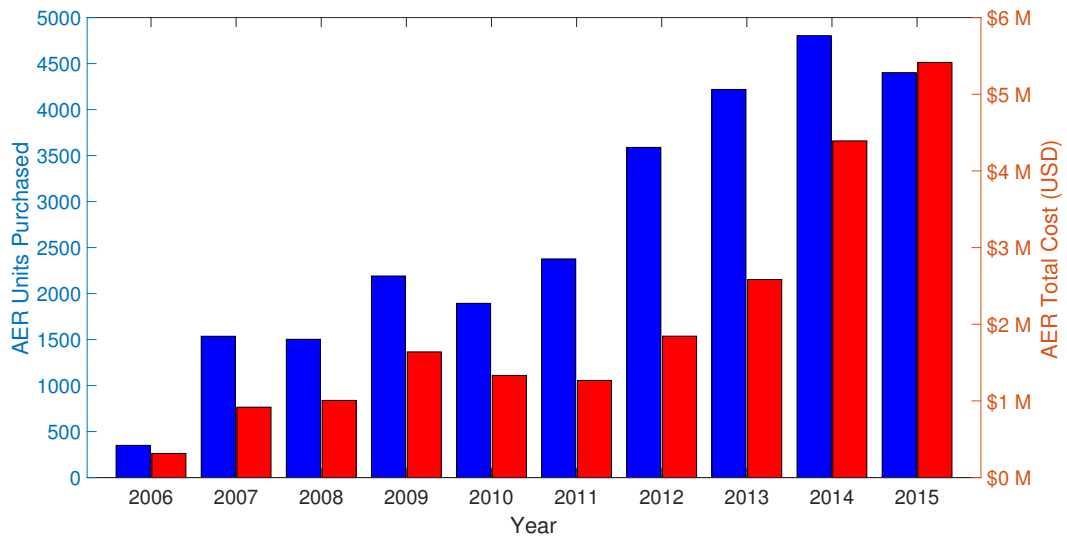


Figure 1: AER Units Ordered and Total Cost by Year

In terms of the number of AER units purchased by the independent risk management firm on behalf of the Company’s insured customers, it is clear that the use of AERs by the Company’s

insured customers as a risk management strategy has been steadily growing since 2005-2006, as demonstrated in Figure 1 above. There were year-over-year increases in the number of AER units purchased in every year from 2010 to 2014. Although the number of AER units purchased on behalf of the Company's insured customers by the independent risk management firm decreased from 2014 to 2015, total expenditures on AERs increased by 23% from roughly \$4.4 M to \$5.4 M during this period.

Figure 2 and Table 2 below break down AER purchase orders with respect to AER vendors.

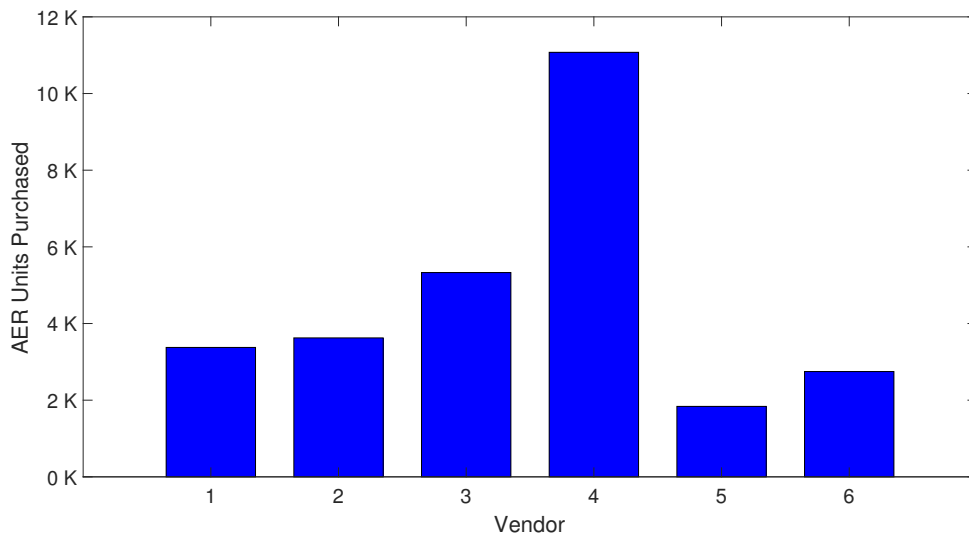


Figure 2: AER Units Ordered by Vendor

Table 2: Breakdown of AER Units, by Vendor, by Year

Year	Vendor1	Vendor2	Vendor3	Vendor4	Vendor5	Vendor6
2006	0.00%	100.00%	0.00%	0.00%	0.00%	0.00%
2007	0.00%	44.90%	0.00%	0.00%	49.70%	5.40%
2008	0.00%	11.20%	0.00%	0.00%	60.30%	28.50%
2009	0.00%	0.30%	26.70%	25.40%	5.00%	42.50%
2010	0.00%	0.00%	16.30%	61.10%	3.10%	19.60%
2011	0.00%	0.00%	26.10%	68.50%	0.00%	5.40%
2012	0.00%	0.90%	18.00%	73.00%	0.00%	8.10%
2013	12.20%	8.80%	18.30%	57.10%	0.00%	3.60%
2014	23.10%	10.90%	40.20%	22.30%	0.00%	3.50%
2015	37.50%	29.20%	7.30%	23.70%	0.00%	2.20%

The names of the AER vendors from which the independent risk management firm purchased AERs on behalf of the Company’s insured customers were made anonymous at the request of the Company.

An analysis of customer performance with respect to AER vendor was begun but ultimately not included in this thesis as sparseness in the data contributed to a failure to produce meaningful or conclusive results.

1.2 Defensive Driving Courses and Training

Defensive driving courses and training are conducted by certified instructors and serve to teach and habituate safe driving techniques to an insured customer’s fleet of drivers. It is a commonly held belief that the participation in defensive driving courses and training by a commercial transportation company has the potential to improve the safety of that company’s fleet, thus preventing or mitigating the damage of auto collisions. Hence, comparing the performance of the subclass of the Company’s insured customers who have participated in defensive driving courses and training to the data’s population class of customers with respect to commercial auto insurance claims frequency and severity is warranted.

From the data provided by the Company, participation in defensive driving courses and training is better represented in the trucking industry (45% of customers) than it is in the passenger transportation industry (8% of customers). However, as defensive driving courses and training should

have a similar potential for reducing claims frequency and severity in the passenger transportation industry as they do in the trucking industry, it is unclear why the data reflects this discrepancy of participation rate by customer industry.

1.3 Physical Abilities Testing (abbr. PAT) Programs

Physical abilities testing programs are designed to ensure that a company's employees are physically capable of performing their occupation's responsibilities. PAT programs are commonly believed to have the potential of preventing workers' compensation insurance claims, as the physical tests of PAT programs are designed to minimize the potential for work-related injuries. This is extremely relevant to companies in the trucking industry, as truck driving is a physically demanding profession. Responsibilities of truck drivers are not limited to merely driving, but often include loading and unloading cargo, cleaning and inspecting trucks, etc. Consequently, it is reasonable that the data reflects that PAT programs are more prevalent in the Company's trucking customers (22% of customers) than in the Company's passenger transportation customers (4% of customers).

The specific physical and sensory tests involved in PAT programs can differ from company to company, and it is unclear to what extent the PAT programs implemented by the Company's insured customers evaluate driving specific abilities such as vision and reaction time. Nevertheless, the mere possibility of PAT programs having a correlation with customer performance with respect to commercial auto insurance claims constitutes grounds for analysis.

1.4 Return-to-Work (abbr. RTW) Programs

Similarly to PAT programs, return-to-work programs are commonly considered a risk management strategy intended to mitigate workers' compensation insurance claims costs as opposed to commercial auto insurance claims costs. The fundamental aim of RTW programs is to expedite the process of returning injured workers to the workplace through practices such as modified job responsibilities during an injured worker's recovery. Thus, the impact of implementing a RTW program does not appear to constitute exorbitant potential with respect to reducing an insured customer's commercial auto insurance claims frequency and severity.

Additionally, the data provided by the Company with respect to insured customers’ use of defensive driving courses and training, PAT programs, or RTW programs does not include information with respect to when or to what extent customers implemented these risk management strategies. Analysis of these risk management strategies will be limited in this regard, and will predominantly focus on the aggregate performance of the customer subclasses. Limitations of the analysis contained in this thesis will be discussed in Subsection 6.1.

2 Literature Review

2.1 Cicchino

With regard to autonomous vehicle technologies that have the potential to be purposed as commercial auto insurance risk management strategies, the main piece of literature that this thesis will leverage is a paper written by the Insurance Institute for Highway Safety’s Jessica Cicchino, titled “Effectiveness of Forward Collision Warning and Autonomous Emergency Braking Systems in Reducing Front-to-Rear Crash Rates” (2016). Cicchino’s study evaluates the effectiveness of two collision avoidance and mitigation technologies that were implemented as optional features by a subset of automakers from 2010 to 2014: **forward collision warning (abbr. FCW)** and **autonomous emergency braking (abbr. AEB)**. Combining crash data from 22 U.S. states during the period of 2010-2014 and associated insurance exposure data from the Highway Loss Data Institute, Cicchino modelled the relationship between a vehicle’s features (i.e. whether or not the vehicle was equipped with FCW or AEB) and the corresponding crash rates as Poisson regressions.

2.1.1 Forward Collision Warning (abbr. FCW) Model

A summary of Cicchino’s FCW model is as follows:

$$\log(\lambda_i) = \beta_0 + \beta_1(f_i) + \beta_2(a_i) + \beta_3(\text{covariates}) ,$$

where $C_i \sim \text{Poisson}(E_i\lambda_i)$ represents the number of crash involvements, E_i represents exposure (in

insured vehicle days), f_i represents the presence or absence of FCW for vehicle i , and a_i represents the presence or absence of FCW with AEB for vehicle i . In the model, $\exp(\beta_1)$ represented “the rate ratio comparing crash involvement rates for vehicles with FCW alone to vehicles without” and $\exp(\beta_2)$ represented “the rate ratio comparing crash involvement rates for vehicles with FCW and AEB to vehicles without” [1].

2.1.2 Autonomous Emergency Braking (abbr. AEB) Model

A summary of Cicchino’s AEB model is as follows:

$$\log(\lambda_i) = \beta_0 + \beta_1(v_{i,1}) + \dots + \beta_n(v_{i,n}) + \beta_{n+1}(\text{covariates}) ,$$

where $C_i \sim Poisson(E_i\lambda_i)$ represents the number of crash involvements, E_i represents exposure (in insured vehicle days), and $v_{i,1}$ through $v_{i,n}$ represent the vehicle model for vehicle i and comparison vehicle model types 1 through n . In the model, $\exp(\beta_n)$ represented the rate ratio comparing crash involvement rates between the AEB-equipped models and comparison vehicle model n .

It should be noted that the AEB-equipped models used for Cicchino’s study were the 2011-2012 model Volvo S60 and the 2010-2012 model Volvo XC60. According to Cicchino, Volvo’s low-speed AEB “can prevent crashes altogether if the speed of a vehicle relative to the speed of the vehicle ahead is 9 mph or less, or it can lessen the severity of the crash by reducing the striking vehicle’s speed if the speed relative to the vehicle ahead is 10-19 mph” [1].

2.1.3 Results and Application

Cicchino defines the term **rear-end striking crash** to include collisions for which “In two-vehicle crashes, a vehicle was the striking vehicle in a rear-end crash if the manner of collision was front-to-rear, no vehicles in the crash were backing, the point of impact on the subject vehicle was the front (11, 12, or 1 o’clock positions), and the point of impact on the struck vehicle was the rear (5, 6, or 7 o’clock positions)” [1]. Given the purpose and functionality of the FCW and AEB technologies, reporting the reduction in rear-end striking crashes for vehicles equipped with FCW

and AEB appears to be an obvious choice of metric.

Cicchino’s study reported that “FCW alone was associated with a 27% reduction in rear-end striking crash rates, low-speed AEB was associated with a 43% reduction, and FCW with AEB was associated with a 50% reduction.” According to Cicchino, the possible 50% reduction in rear-end striking crashes associated with equipping all vehicles with FCW and AEB would have corresponded to a prevention of 1 million U.S. police-reported rear-end crashes and over 400,000 related injuries in 2014 [1].

With regard to the availability of FCW and AEB technologies in the near future, Cicchino comments “Twenty automakers representing 99% of the U.S. auto market have committed to making FCW and AEB standard features on virtually all new passenger vehicles by 2022” [1]. Although automakers have taken the initiative to steadily implement these technologies in passenger vehicles as new models roll out every year, the influx of these technologies has been less apparent in the commercial vehicle sector. Cicchino’s model of the effectiveness of FCW and AEB in preventing and mitigating rear-end striking crashes will be applied to the Company’s data to ascertain the potential for these technologies as risk management strategies for the Company’s insured customers. Discussion of the potential of these technologies if purposed as risk management strategies can be found in Subsection 7.1.

3 Methods

3.1 Performance Metrics

The Company-standard metrics for reporting commercial auto insurance claims frequency will be imported to the analysis of this thesis, and is described below in Subsection 3.1.1. Additionally, this thesis introduces a new standardized metric with regard to tracking the average total incurred cost of claims associated with an insured customer, which can be found in Subsection 3.1.2.

3.1.1 Frequency

The standard metric used by the Company to report an insured customer's commercial auto claims frequency in the trucking industry is given in terms of the number of commercial auto insurance claims per one million miles driven by the insured customer's fleet. That is to say, the frequency of claims f_i^{truck} for a trucking customer in policy period i can be represented by the following equation:

$$f_i^{truck} = \frac{\sum c_i * 1\,000\,000}{\sum m_i},$$

$\sum c_i$:= the sum of a customer's commercial auto insurance claims in policy period i

$\sum m_i$:= the sum of miles driven by a customer's entire fleet in policy period i .

With regard to the passenger transportation industry, the Company's standard for reporting commercial auto insurance claims frequency involves the number of *units* that an insured customer's fleet records in a given policy period. Here, the term **unit** represents a measure corresponding to the number of vehicles that a customer's fleet operates in a given policy period, adjusted by the Company using a proprietary method. The frequency of claims $f_i^{passenger}$ for a passenger transportation customer is given by the number of commercial auto insurance claims per 100 units recorded in a given policy period, and is represented by the following equation:

$$f_i^{passenger} = \frac{\sum c_i * 100}{\sum u_i},$$

$\sum c_i$:= the sum of a customer's commercial auto insurance claims in policy period i

$\sum u_i$:= the sum of units recorded by a customer's entire fleet in policy period i .

3.1.2 Average Severity

The term **average severity** is introduced in this thesis to denote a standardized metric tracking the average *total incurred cost* of claims associated with a particular insured customer in a given policy period. With regard to an individual commercial auto insurance claim, the term **total incurred cost** equates to the sum of the total amount paid on a claim and the total amount set aside in reserves. For the analysis of this thesis, average severity will be reported in terms of the sum of total incurred costs of an insured customer’s commercial auto insurance claims per one million miles driven and one hundred units recorded for the trucking and passenger transportation industries, respectively. See the following two equations immediately below:

$$\bar{s}_i^{truck} = \frac{\sum T_i * 1\,000\,000}{\sum m_i},$$

$\sum T_i$:= the sum of total incurred costs of a customer’s commercial auto claims in policy period i

$\sum m_i$:= the sum of miles driven by a customer’s entire fleet in policy period i .

$$\bar{s}_i^{passenger} = \frac{\sum T_i * 100}{\sum u_i},$$

$\sum T_i$:= the sum of total incurred costs of a customer’s commercial auto claims in policy period i

$\sum u_i$:= the sum of units recorded by a customer’s entire fleet in policy period i .

The average severity metric introduced in this subsection differs from the intuitive “cost per claim” metric, which tracks the sum of total incurred costs with respect to the number of claims an insured customer files in a given policy period. Reporting average severity with respect to miles driven (in trucking) and units recorded (in passenger transportation) better standardizes the metric with respect to an insured customer’s risk exposure, which is critical when evaluating performance.

3.2 Statistical Analysis

3.2.1 Standard Error of the Mean

In the figures contained in Subsections 5.2, 5.3, and 5.4, frequency and average severity means are reported for an insured customer subclass by policy year with their associated standard errors, for which computation is given by the equation below:

$$SE_{\bar{x}_{k,i}} = \frac{s_{k,i}}{\sqrt{n_{k,i}}},$$

$s_{k,i}$:= the standard deviation from insured customer subclass k in policy year i

$n_{k,i}$:= the number of observations from insured customer subclass k in policy year i .

3.2.2 Two-Sample t -Test

In Subsection 5.5, the frequency and average severity metrics are aggregated across all policy years. The primary method used in this thesis for comparing these aggregate frequency and average severity means from two different insured customer subclasses is the two-sample t -test, for which confidence intervals bounds are given by the expression below [2]:

$$(\bar{x}_b - \bar{x}_k) \pm t_{1-\alpha/2,v} \sqrt{\frac{s_b^2}{n_b} + \frac{s_k^2}{n_k}},$$

\bar{x}_b := the mean for the **baseline** insured customer class, represented by the total data population

s_b^2 := the variance of the observations from the baseline insured customer class

n_b := the number of observations from the baseline insured customer class

\bar{x}_k := the mean for insured customer subclass k , determined by risk management strategy use

s_k^2 := the variance of the observations from insured customer subclass k

n_k := the number of observations from insured customer subclass k

$t_{1-\alpha/2,v}$:= the t -distribution critical value at significance level α with v degrees of freedom.

When the performance metrics (frequency, average severity) are averaged across all policy years and across all insured customers in a particular subclass, the number of data observations becomes large enough to assume infinite degrees of freedom. The selection of t -distribution critical values for the statistical test reflects this feature of the data, and consequently the comparison of means is equivalent to a two-sample z -test. Confidence interval bounds computed for the comparison of means are included in Appendix A.2

3.3 AER Analysis Model

3.3.1 Year Rank

AER purchase order histories associated with the Company’s insured customers spanned eleven insurance policy years (2006-2016). For a given insured customer, these AER purchase order histories are generally scattered across several disjoint policy years. In order to identify the policy years for which the use of AERs as a risk management strategy would have the highest impact on customer performance, the term **year rank (abbr. YR)** is introduced in this thesis to denote the policy years during which a customer experienced the largest volume of AER units purchased. For example, a customer’s policy year with an associated year rank of 1 corresponds to the policy year during which the customer purchased the most AER units. A customer’s policy year with an associated year rank of 2 corresponds to the policy year during which the customer purchased the second-most AER units, and so on.

Table 3: Aggregate Year Rank Data

Year Rank	AER Units	Percentage
1	15,046	56%
2	6,051	22%
3	2,609	10%
4	1,661	6%
5	837	3%
6	373	1%
7	181	1%
8	88	0%
9	78	0%

Table 3 shows the quantity and percentage of AER units purchased on behalf of the Company’s insured customers with respect to year rank. Note that year ranks 1-3 together account for 88% of all AER unit purchases. As a modelling simplification, policy periods with year ranks 1-3 were assumed to be the only years having a substantial impact on an AER customer’s performance in the subsequent two-year lag time.

3.3.2 Two-Year Lag Time

When an AER purchase order is entered through an associated vendor, there is a variable amount of time between when the purchase order was entered and when the AER units are delivered to the insured customer. Additionally, there is a separate variable amount of time from when the AER units are delivered to the insured customer and when the units are installed in the customer’s fleets, thus having the potential to improve customer performance.

After speaking with the Company’s Director of Risk Management, a simple and effective policy for accounting for this time delay between AER purchase order dates and subsequent AER unit installation is to assume that AER units ordered in policy year i have the largest impact on the frequency and average severity of the associated customer’s commercial auto claims in policy years $i + 1$ and $i + 2$. Throughout this thesis, this assumption will be referred to as the **two-year lag time** assumption.

When examining the performance of an insured customer using AERs as a risk management strategy, only policy years with year ranks 1-3 and the subsequent two-year lag time were used to measure the change in the customer’s performance.

4 Commercial Auto Claims Overview

This section of the thesis serves to provide a high-level overview of the over 25,000 anonymized commercial auto insurance claims included in the data provided by the Company. Claims were examined with respect to customers’ U.S. state of origin (Subsection 4.1), the distribution of the total incurred costs of claims (Subsection 4.2), and claim descriptions (Subsection 4.3).

4.1 Customer Location

Table 4: Claims by Customer's U.S. State of Origin

Customer State	Claim Count	Percentage
NY	3,687	14.60%
CA	3,518	13.90%
TX	1,896	7.50%
MA	1,668	6.60%
NJ	1,665	6.60%
PA	1,313	5.20%
IL	928	3.70%
WI	916	3.60%
FL	810	3.20%
CT	713	2.80%

Claims are attributed to U.S. states according to the state for which the associated customer's policy was written. Table 4 lists the top ten U.S. states responsible for the most claims. To clarify, Table 4 signifies that 14.6% of the claims analyzed in this thesis are attributed to customers located in New York, that 13.9% of the claims are attributed to customers located in California, and so on. The top ten states listed in Table 4 together account for 17,114 of the 25,256 claims, or roughly 68% of the total claims.

The anonymized claims data provided by the Company appears to be geographically representative of the entire United States, as there is high correlation between the number of customers' claims associated with a U.S. state and the state's population. Of the ten U.S. states listed in Table 4, six constitute the states with the largest populations, according to the 2010 U.S. Census. In order, these states are: (1) CA, (2) TX, (3) NY, (4) FL, (5) IL, and (6) PA [3]. That is to say, geographic bias in the Company's claims should not be considered a significant limitation to the analysis of this thesis.

4.2 Total Incurred Cost of Claims

Figure 3 below provides a histogram of total incurred costs for all of the claims in the Company's data.

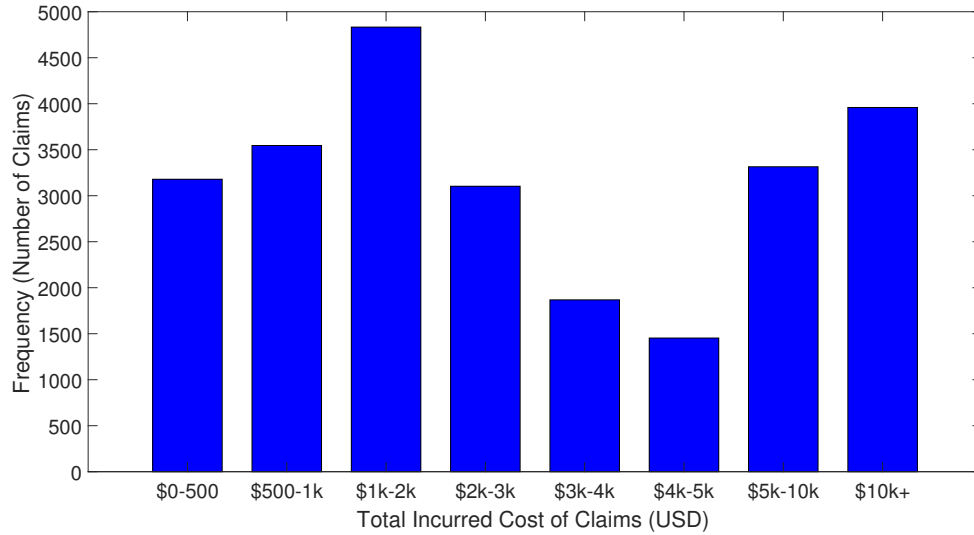


Figure 3: Histogram of Total Incurred Costs

84% of the claims provided in the Company’s data (21,297 of 25,256 claims) have total incurred costs less than or equal to \$10k. Table 5 below breaks down the right tail of this distribution with respect to quantiles.

Table 5: Total Incurred Cost Quantiles

Total Incurred Cost	Quantile
\$10,000	84.30%
\$20,000	91.70%
\$30,000	94.30%
\$40,000	95.60%
\$50,000	96.40%
\$60,000	96.90%
\$70,000	97.30%
\$80,000	97.60%
\$90,000	97.80%
\$100,000	98.00%
\$500,000	99.50%
\$1,000,000	99.80%

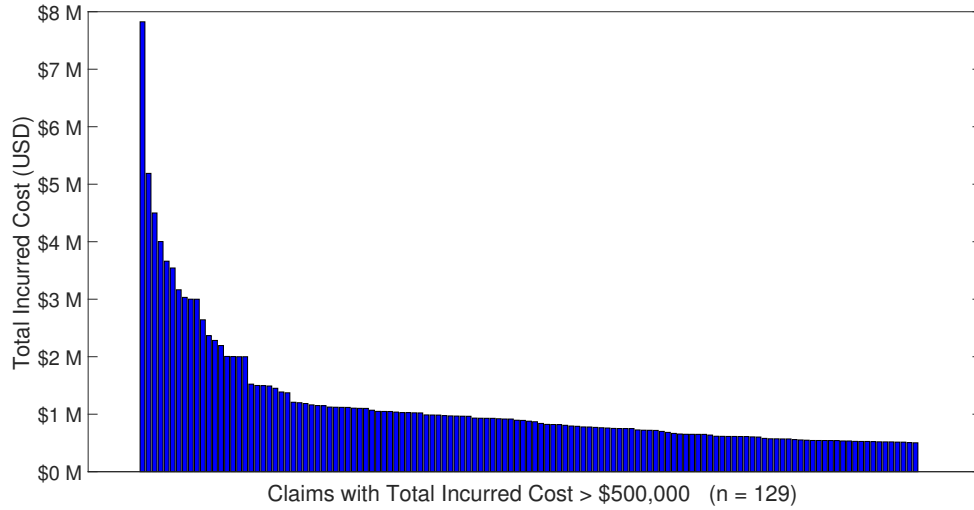


Figure 4: Total Incurred Distribution Tail

Figure 4 provides a visual representation of the distribution’s tail. Additionally, It should be noted that the data pertaining to the total incurred cost of the Company’s claims used in this thesis may not be completely accurate as 8.2% of the claims (2,068 of 25,256) are listed as having a status of “Open,” thus allowing for future developments in the claims. Implications of this incompleteness in the data will be discussed in Subsection 6.1.

4.3 Claim Descriptions

Each claim provided by the Company contains a field describing the cause of the associated collision. Table 6 below lists the top ten descriptions associated with claims provided by the Company.

Table 6: Commercial Auto Claims by Description

Claim Description	Percentage of Claims
INSURED VEHICLE HIT OTHER VEHICLE	14.30%
INSURED REAR-ENDED CLAIMANT	8.40%
INSURED VEHICLE HIT PARKED OTHER VEHICLE	5.20%
OTHER VEHICLE HIT INSURED VEHICLE	4.30%
INSURED VEHICLE BACKED INTO OTHER VEHICLE	3.80%
INSURED VEHICLE HIT OBJECT	3.00%
INSURED VEHICLE SIDESWIPE OTHER VEHICLE	1.80%
RECORD ONLY	1.60%
CLAIMANT REAR-ENDED INSURED	1.50%
INSURED VEHICLE STRUCK OTHER VEHICLE	1.30%

Claim descriptions such as “Insured vehicle hit other vehicle” and “Other vehicle hit insured vehicle” are inherently unhelpful for any sort of data analysis. These descriptions fail to include key features of the associated collision and consequently provide no value with regard to discovering the root causes of the Company’s insured customers’ collisions.

However, attention should be drawn to the claim description with the second-highest frequency: “Insured rear-ended claimant.” Given the vagueness of several of the top ten claim descriptions and the potential for claims to be misclassified by description, the 8.40% statistic representing the percentage of rear-end collisions caused by one of Company’s customers is interpreted as a lower bound for the percentage of these types of collisions. This statistic will be used to predict the potential effectiveness of forward collision warning and autonomous emergency braking as risk management strategies in Subsection 7.1.

5 Results

Subsection 5.1 analyzes the performance of the subclass of insured customers who have used AERs as a risk management strategy. Analysis is broken down by the following two factors: the size of a customer’s fleet, and whether or not an insured customer’s implementation of the AERs was judged as best-in-class or worst-in-class by risk management specialists.

Subsections 5.2, 5.3, and 5.4 each follow an identical methodology and examine the performance

of customers who implemented defensive driving courses and training, PAT programs, and RTW programs, respectively. The figures in these sections compare the year-to-year frequency and average severity means of these customer subclasses with a baseline customer class, for which the total population of customers contained in the Company’s data was selected. All means are displayed with an error bar which reflects one standard error of the mean (for which computation was outlined in Subsection 3.2.1).

Subsection 5.5 aggregates performance metrics data across all policy years, and compares the frequency and average severity means for the following customer subclasses: defensive driving courses and training, PAT programs, RTW programs, and the baseline customer class. Two-sample *t*-tests were used to compare the customer subclass performance metric means with the baseline customer class performance means at three different significance levels: $\alpha = 0.05$, $\alpha = 0.10$, and $\alpha = 0.20$.

5.1 Automated Event Recorders (abbr. AERs)

5.1.1 Performance by Insured Customer Fleet Size

The sizes of insured customers’ fleets are measured in terms of the number of units recorded in a given policy period (the term *units* was introduced in Subsection 3.1.1). The analysis of this section of the report is confined to the passenger transportation industry, as insured customers in the trucking industry do not generally use units as a reporting standard. In the passenger transportation industry, the Company’s customers comprise 17 small fleets (1-20 units), 47 medium-sized fleets (20-50 units), and 104 large fleets (50+ units).

Table 7 displays the change in claims frequency from policy years with year ranks 1-3 through the associated two-year lag times. The column “YR Frequency” represents a fleet size customer subclass’s average claims frequency at the time when the most AER units were purchased by insured customers. The column “Lag Time Frequency” represents that subclass’s average claims frequency in the subsequent two policy years.

Table 7: Changes in Claims Frequency by Fleet Size

Fleet Size	YR Frequency	Lag Time Frequency	Percent Change
Small	9.78	8.90	-9.00%
Medium	10.18	13.46	32.22%
Large	7.55	7.99	5.83%
All Fleet Sizes	7.66	8.21	7.18%

The year rank model suggests that the only fleet size customer subclass to exhibit a decrease in claims frequency after implementing AERs as a risk management strategy was small fleets (1-20 units), with a claims frequency reduction of 9%. For a breakdown of changes in frequency by year rank for each fleet size customer subclass, see Appendix A.1.

5.1.2 Performance by Best-in-Class and Worst-in-Class AER Implementations

Three associates from the Company’s risk management department were interviewed and asked to subjectively identify the best-in-class and worst-in-class AER implementations, given their knowledge of the insured customers who have used AERs as a risk management strategy. Of the customers who had used AERs as a risk management strategy, 19 customers were identified as having best-in-class AER program implementations and 22 customers were identified as having worst-in-class AER program implementations. These customers span two industries that dominate the Company’s book of business: passenger transportation (abbr. “passenger” in some subsequent figures and tables), and trucking (abbr. “truck” in some subsequent figures and tables). The breakdown of customers with the best-in-class and worst-in-class AER implementations by industry is included below in Table 8.

Table 8: Best-in-Class and Worst-in-Class AER Implementations, by Industry

Customer Classification	Passenger	Truck
Best-in-Class AER Implementations	7	12
Worst-in-Class AER Implementations	8	14

Tables 9 and 10 included below give the change in claims frequency from policy years with year ranks 1-3 through the subsequent two-year lag times.

Table 9: Frequency Changes by AER Implementation Classification: Passenger Transportation

Customer Classification	YR Frequency	Lag Time Frequency	Percent Change
Passenger Best-in-Class	3.52	3.67	4.03%
Passenger Worst-in-Class	7.22	6.58	-8.90%

Table 10: Frequency Changes by AER Implementation Classification: Trucking

Customer Classification	YR Frequency	Lag Time Frequency	Percent Change
Truck Best-in-Class	0.99	1	0.88%
Truck Worst-in-Class	1.78	1.52	-14.75%

The year rank model suggests that there is a significant decrease in claims frequency associated with the worst-in-class AER implementations for both the passenger transportation and trucking industries. Best-in-class AER implementations appear to have had relatively little impact on changes in claims frequency reflected in the year rank model, corresponding to roughly 4% and 0.9% claims frequency increases in the passenger transportation and trucking industries, respectively.

In addition to applying the year rank model to the customer subclasses of best-in-class and worst-in-class AER implementations, these subclasses were analyzed with respect to overall customer performance regardless of when AER units were purchased. Figures 5 and 6 display customer subclass performance metrics averaged across all policy years. The customer subclass referred to as “All Other AER” denotes the Company’s insured customers who have used AERs as a risk management strategy but were not classified as having either best-in-class or worst-in-class AER implementations. The “All Other AER” and “Non-AER” customer subclasses are included in Figures 5 and 6 to serve as baseline performance standards.

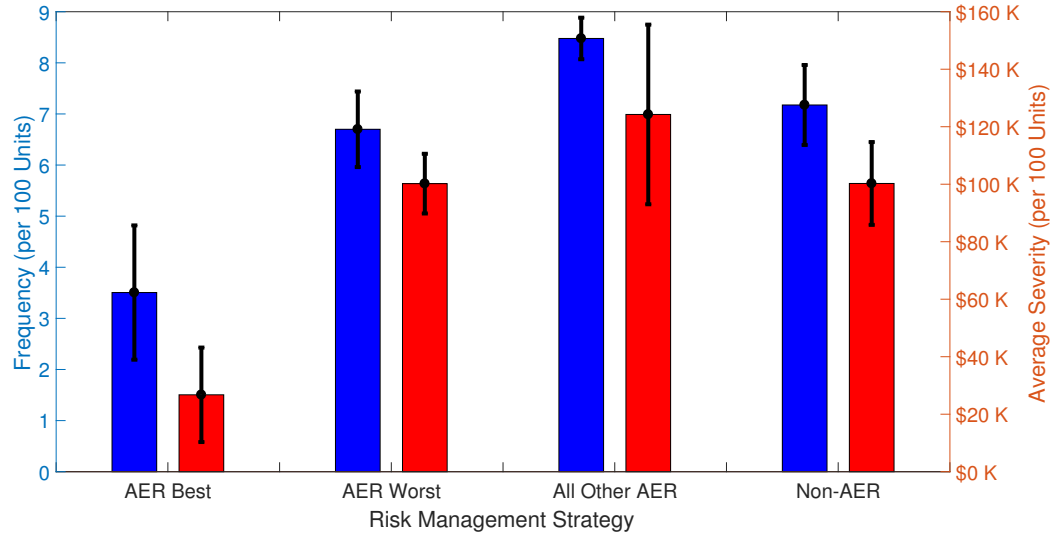


Figure 5: Best and Worst Implementations: Passenger

Figure 5 demonstrates that customers in the passenger transportation industry with AER best-in-class implementations are shown to have significantly lower claims frequencies and average severities than the baseline performance standards irrespective of when AER units were purchased. In fact, the mean frequency reported for customers with best-in-class AER implementations (3.51 claims per 100 units recorded) is less than half of the frequency means from either the “All Other AER” subclass (8.48 claims per 100 units) or the “Non-AER” subclass (7.18 claims per 100 units).

In contrast, customers in the passenger transportation with worst-in-class AER implementations cannot be shown to exhibit higher claims frequencies or higher average severities than the baseline performance standards.

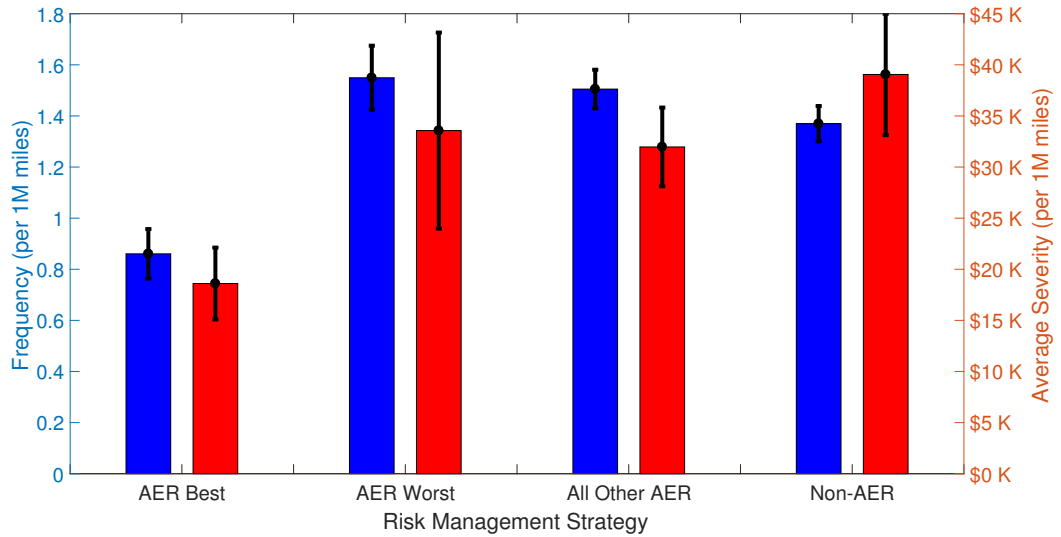


Figure 6: Best and Worst Implementations: Truck

Similar to the passenger transportation industry results, there exists a correlation between a customer in the trucking industry having a best-in-class AER implementation and customer performance superior to that of baseline performance standards. However, it should be noted that this observed correlation in the data does not imply causation. That is to say, Figures 5 and 6 do *not* support the claim that AER best-in-class implementations are a direct cause of outperforming baseline performance standards. Figures 5 and 6 will be referenced in Section 6 to discuss the possibility of selection bias when classifying customers as best-in-class or worst-in-class AER implementations.

5.2 Defensive Driving Courses and Training

Considering the subclass of customers who participated in defensive driving courses and training, Figures 7 and 8 display the year-by-year mean frequencies and mean average severities for the passenger transportation and trucking industries, respectively.

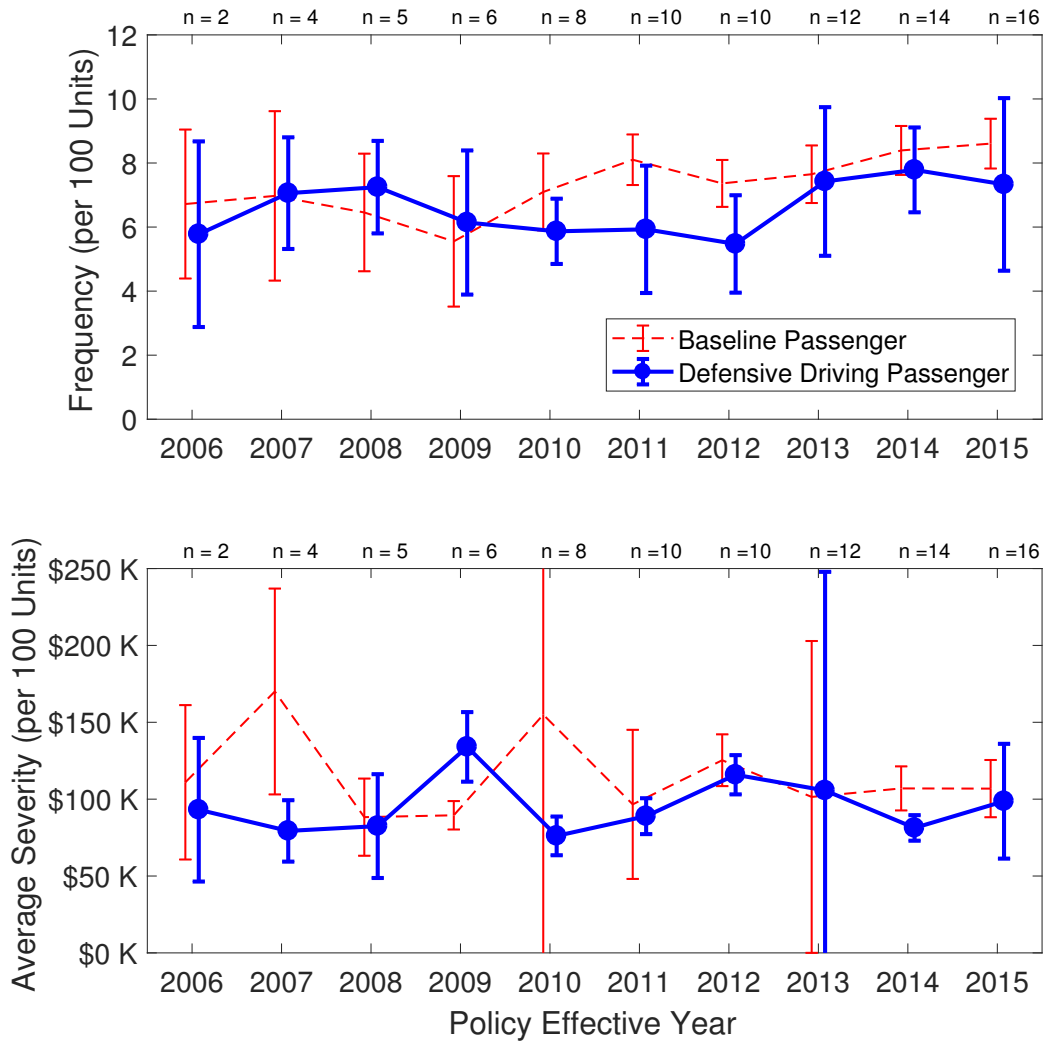


Figure 7: Defensive Driving Subclass Performance: Passenger Transportation

For the passenger transportation industry, Figure 7 shows that neither the frequency nor the average severity means of the defensive driving subclass can be shown to consistently outperform the baseline performance standards on a year-by-year basis.

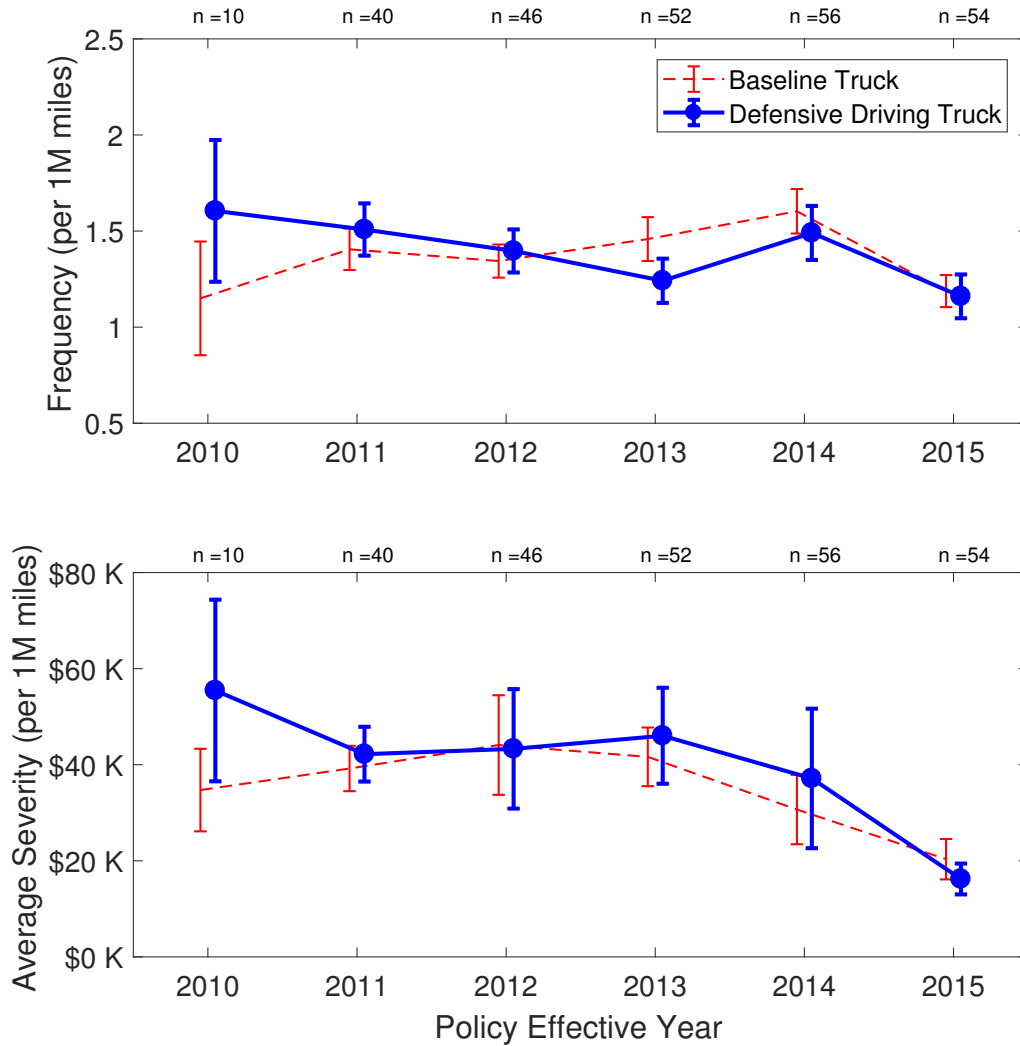


Figure 8: Defensive Driving Subclass Performance: Trucking

Similarly in the trucking industry, insured customer frequency and average severity means in the defensive driving subclass cannot be shown to outperform the baseline standards on a year-by-year basis. As demonstrated in Figure 8, there exists consistent overlap between one standard error of the means for the defensive driving subclass and the baseline performance standards.

5.3 Physical Abilities Testing (abbr. PAT) Programs

Considering the subclass of customers who implemented PAT programs as a risk management strategy, Figures 9 and 10 display the year-by-year mean frequencies and mean average severities for the passenger transportation and trucking industries, respectively.

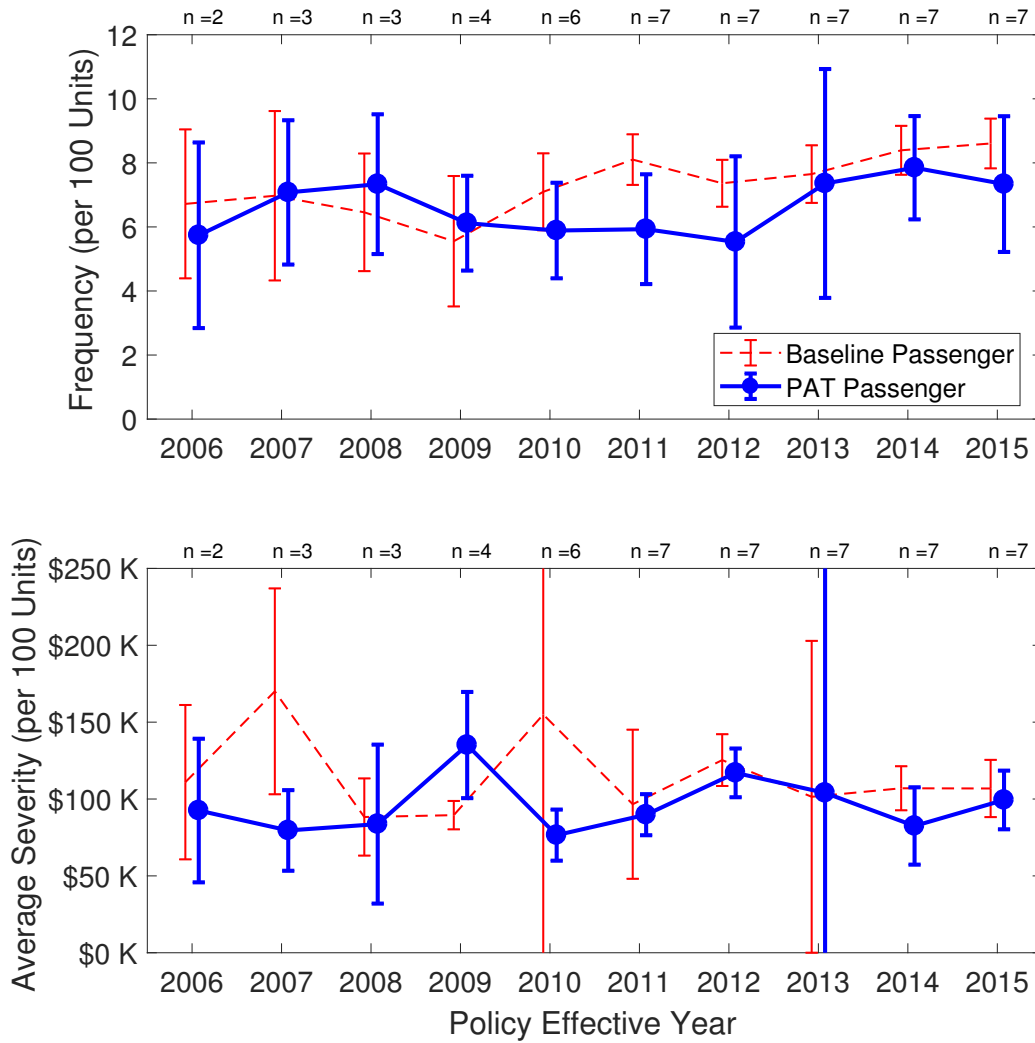


Figure 9: PAT Program Subclass Performance: Passenger Transportation

Implementation of a PAT program cannot be shown to correlate with customers outperforming the baseline performance standards in the passenger transportation industry. When accounting for one standard error of the means reported, Figure 9 reveals that the PAT passenger transportation subclass does not outperform the baseline standards in any policy year for either metric.

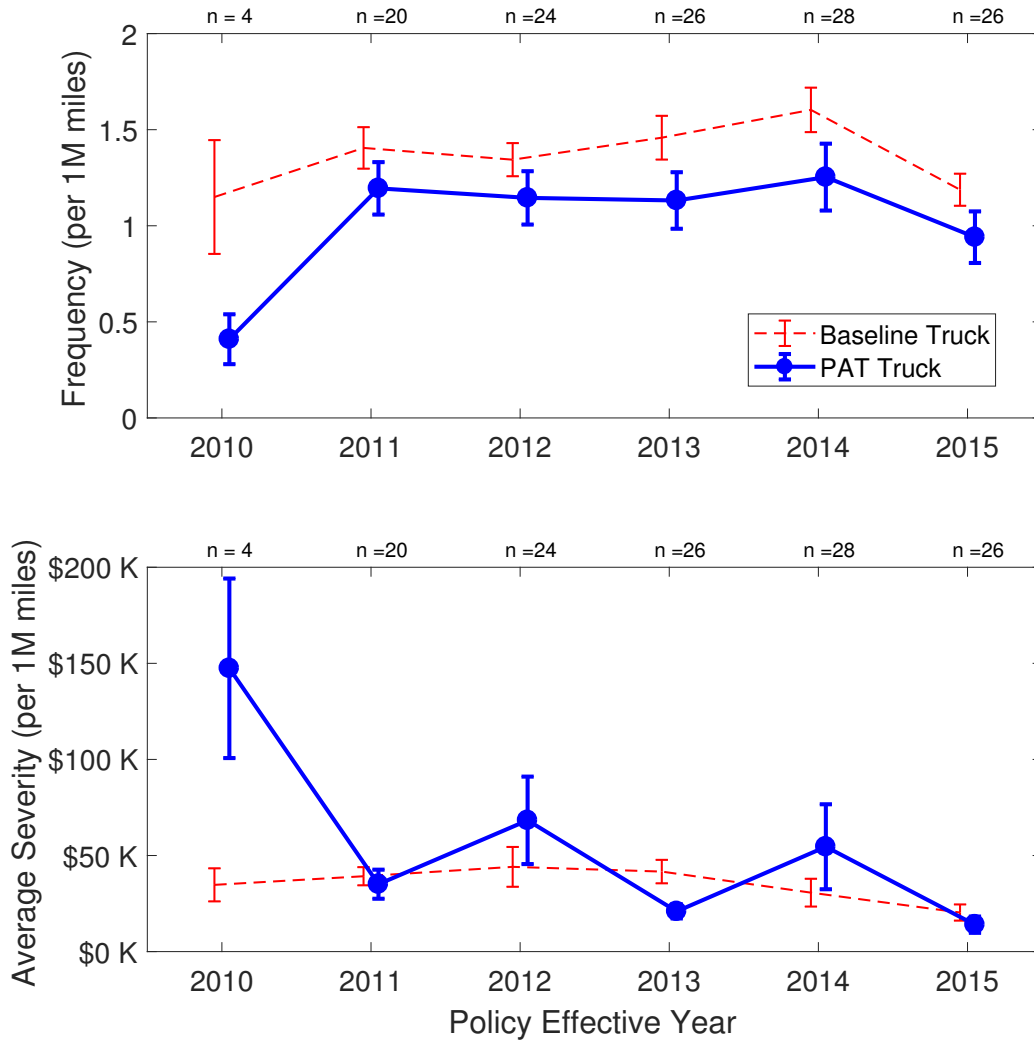


Figure 10: PAT Program Subclass Performance: Trucking

In the trucking industry, the PAT subclass demonstrates a lower claims frequency mean than the baseline customer class in every policy year examined. In four out of six of these policy years, Figure 10 confirms the difference of these frequency means within one standard error of the means reported.

Although there appears to be a correlation between implementing a PAT program in the trucking industry and lower claims frequency, this correlation does not appear to extend to the average severity of the subclass. In three out of the six policy years observed, the PAT subclass exhibited higher average severities than the baseline performance standard.

5.4 Return-to-Work (abbr. RTW) Programs

Considering the subclass of customers who implemented RTW programs as a risk management strategy, Figures 11 and 12 display the year-by-year mean frequencies and mean average severities for the passenger transportation and trucking industries, respectively.

Passenger transportation customers in the RTW subclass experienced lower frequency means than the baseline standards in every policy year after 2009. However, Figure 11 reveals that none of the differences of the associated means are confirmed for these policy years when accounting for one standard error of the means reported.

Additionally, the reported average severities for the passenger transportation RTW subclass were lower than the baseline standards in eight out of the ten policy years analyzed. However, Figure 11 casts doubts on this correlation as none of the eight instances of lower means in the RTW subclass are confirmed within one standard error of the means reported.

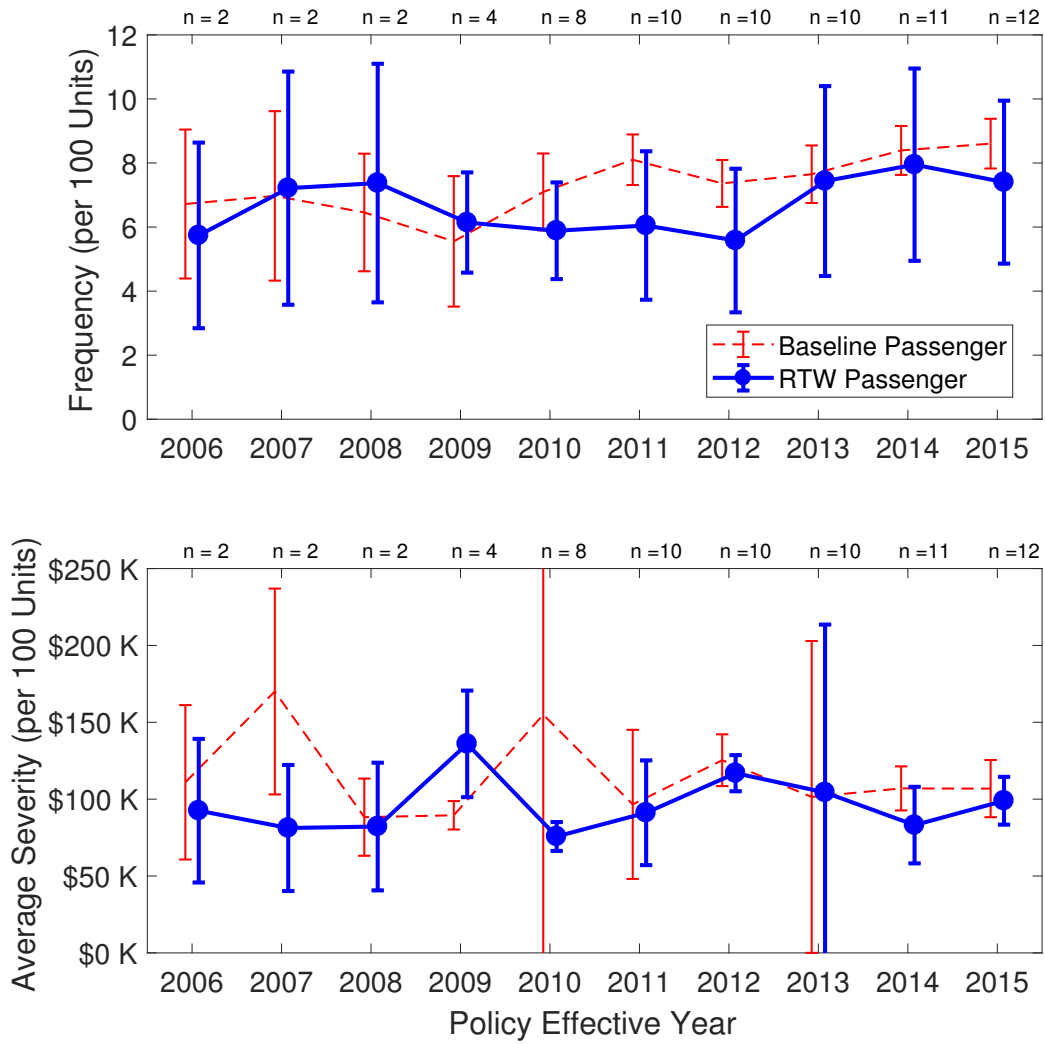


Figure 11: RTW Program Subclass Performance: Passenger Transportation

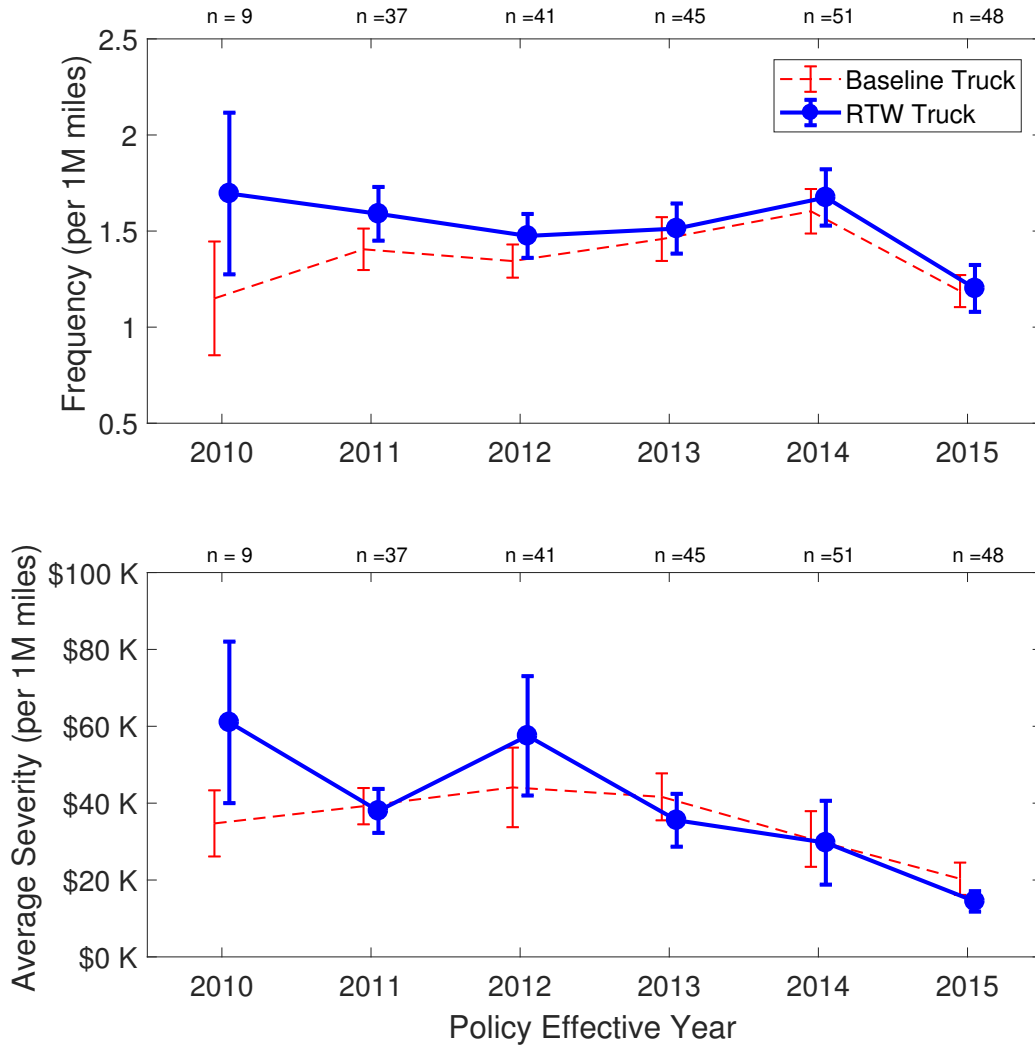


Figure 12: RTW Program Subclass Performance: Trucking

In contrast to the results of the passenger transportation industry, the RTW subclass in the trucking industry reported higher frequency means in every policy year analyzed. Although, as shown in Figure 12, these differences are not confirmed within one standard error of the means reported.

With regard to average severity, Figure 12 suggests neither consistently better or worse performance when comparing the RTW trucking subclass to the baseline performance standards.

5.5 Summary

Figures 13 and 14 display the frequency and average severity means averaged across all policy years. Confidence intervals were computed at the 95%, 90%, and 80% confidence levels to determine if the performance means of any of the risk management strategy subclasses were significantly better or worse than the baseline performance standards, and are included in Appendix A.2.

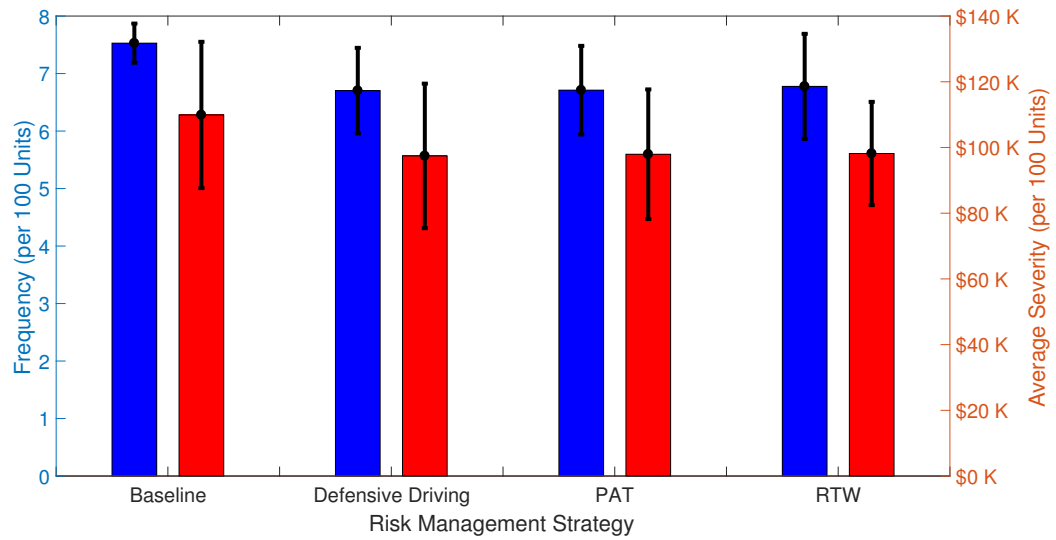


Figure 13: Average All-Time Frequencies: Passenger

Considering both performance metrics within the defensive driving, PAT, and RTW subclasses of the passenger transportation industry data, differences between any of the subclass means and the associated baseline standards cannot be confirmed at the 95%, 90%, or 80% confidence levels. One possible reason for confidence intervals failing to confirm differences between the subclass means and baseline standards in the passenger transportation industry is sparseness in the data, which will be discussed in Subsection 6.1.

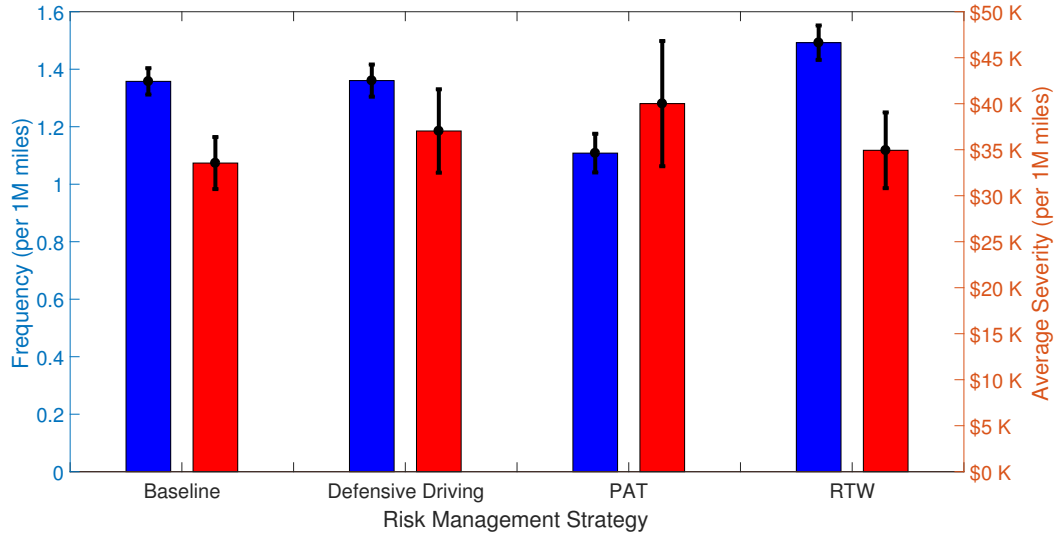


Figure 14: Average All-Time Frequencies: Truck

In contrast to the results of the passenger transportation industry, the confidence intervals computed for the trucking industry data confirm differences between strategy subclass frequency means and baseline standards for two different risk management strategies. For the PAT subclass, the 95% confidence interval for the difference of frequency means given by $\bar{x}_b - \bar{x}_{PAT}$ is bounded by the following interval: $[0.09 \ 0.41]$. That is to say, from the Company’s data, it can be said with 95% confidence that the trucking PAT subclass outperforms the baseline frequency standard by at least 0.09 claims per one million miles driven.

Customers of the RTW subclass can surprisingly be shown to exhibit worse frequencies than the baseline standards. The 90% confidence interval for the difference of frequency means given by $\bar{x}_b - \bar{x}_{RTW}$ is bounded by the following interval: $[-0.26 \ -0.01]$. That is to say, the RTW subclass can be said to exhibit frequencies worse than the baseline standards by at least 0.01 claims per one million miles driven. This confirmation by the 90% confidence interval implies that the average claims frequency of the RTW trucking subclass is pure dreck, or rubbish.

6 Discussion

This thesis' evaluation of the use of AERs as a risk management strategy was unique when compared to the evaluation of the other strategies in that there existed data pertaining to when and to what extent the Company's insured customers used AERs. In the evaluation of AERs, the year rank model and the two-year lag time assumption were introduced as an attempt to account for the element of time. For defensive driving courses and training, PAT programs, and RTW programs, there was no data available with regard to when specifically insured customers implemented these strategies. See Subsection 6.1 for further discussion with regard to limitations in the analysis of these strategies.

In the breakdown of AERs by fleet size in the passenger transportation industry in Subsection 5.1.1, the results showed that the only fleet size to experience a decrease in claims frequency in the two-year lag time was small fleets of 1-20 units (see Table 7). Additionally, medium-sized fleets of 20-50 units appeared to exhibit significantly poor performance in the two-year lag time, as average frequencies increased by 32%. Although this observed correlation in the data does not imply causation, there are circumstances which could attribute to this phenomenon.

One possible explanation is that customer performance is linked with a customer's quality of their AER implementation, which could be associated with customer fleet size. In small fleets, reviewing AER video logs for an entire fleet is commonly believed to be neither an extremely time-intensive nor labor-intensive task. In large fleets, it is commonly believed that most customers who use AERs have a streamlined process for reviewing AER video logs and correcting poor driver habits. Thus, it is possible that medium-sized fleets exist as fleets large enough such that reviewing AER video logs is a nontrivial task, but do not have the efficiency of streamlined processes observed in large fleets.

The proposition that customer performance is a result of the quality of a customer's AER implementation was examined in Subsection 5.1.2. However, perhaps counter-intuitively, the results in Tables 9 and 10 suggest that worst-in-class AER implementations have a higher correlation with exhibiting lower claims frequency in the two-year lag time than best-in-class AER implementations. Worst-in-class AER implementations were shown to correlate with 8.9% (passenger transportation)

and 14.8% (trucking) decreases in claims frequencies in the two-year lag time, whereas best-in-class AER implementations were correlated with 4.0% (passenger transportation) and 0.9% (trucking) increases in claims frequencies in the two-year lag time.

The results in Tables 9 and 10 appear to contradict the intuition that the quality of a customer’s AER implementation is correlated with the perceived change in customer performance after implementing AERs. However, it should be noted that the process of recognizing best-in-class and worst-in-class AER implementations may be subject to selection bias, as the risk management specialists interviewed were generally aware of historical customer performance when categorizing AER implementations as best-in-class or worst-in-class. Indeed, Figures 5 and 6 confirm that there is a significant correlation between a customer being recognized as having a best-in-class AER implementation and performing significantly better than baseline standards irrespective of when AER units were purchased. That is to say, the best-in-class and worst-in-class AER implementation classifications may be simply more indicative of the overall historical performance of a customer than the quality of that customer’s AER implementation.

If this is the case, it should not come as much of a surprise that the “worst-in-class” classified customers experienced decreases in claims frequency while the “best-in-class” classified customers did not experience a decrease in claims frequency after implementing AERs. Intuition dictates that the customers with the worst historical performances have a higher potential for performance improvement by implementing AERs (regardless of the quality of the implementation) than customers with the best historical performances. That is to say, the use of AERs as a risk management strategy appears to be most significantly correlated with improving the performance of historically poor performing customers, and appears to have little to no impact on historically well performing customers.

6.1 Limitations

As mentioned earlier in this section, analysis of defensive driving courses and training, PAT programs, and RTW programs as risk management strategies is limited by the lack of data pertaining to when customers implemented these strategies. Modelling assumptions similar to the year rank

model and two-year lag time assumption used in the analysis of AERs would ideally be used for these strategies to examine customer performance with respect to before and after these strategies were implemented. However, given that time-specific data was not available for the defensive driving, PAT, and RTW subclasses of customers, correlations in customer performance was confined to comparing these subclasses against baseline performance standards on a year-to-year basis.

An additional limitation to the analysis of defensive driving courses and training, PAT programs, and RTW programs as risk management strategies is the number of data observations, particularly in the passenger transportation industry. In Figures 7, 9, and 11, the number of observations n is included at the top of each figure on a year-to-year basis. Note that in the passenger transportation industry, the defensive driving, PAT, and RTW subclasses rarely contain values of n greater than $n = 10$ for a given policy year. Consequently, it should not be surprising that any of the passenger transportation confidence intervals failed to confirm that the means of these subclasses were different from the baseline performance standards. Given such small sample sizes, the variances in the data are simply too great to produce conclusive results.

When examining the average severity performance metric, it should be noted that the reported figures in this thesis are not completely accurate as some of the claims are classified as “Open” and are subject to new developments. As noted in Subsection 4.2, 8.2% of the claims (2,068 of 25,256) from the Company’s data are listed as having the status “Open.” Of the 2,068 open claims, 52% of the open claims are from the policy year 2015, and 16% of the open claims are from the policy year 2014. That is to say, if the average severity metric is skewed due to open claims, it will be skewed in that it underestimates the total incurred cost of claims predominantly within the last two policy years.

7 Conclusion

From the analysis of AERs as a risk management strategy in Subsection 5.1, the most conclusive result appeared to be the correlation between worst-in-class AER implementations and improved claims frequency by 8.9% and 14.8% in the passenger transportation and trucking industries, re-

spectively. As discussed in Section 6, the classification of “worst-in-class” AER implementation is more or less shorthand for “historically poor performing customer.” A reasonable recommendation for the Company would be to clearly identify historically poor performing insured customers and prioritize getting these customers to implement AERs as a risk management strategy. Little to no effort should be spent on getting historically well performing customers to implement AERs as a risk management strategy.

In Subsection 5.5, it was determined that the only risk management strategy subclass to outperform baseline standards was the PAT subclass of the trucking industry, in which claims frequency was shown to be at least 0.09 claims per million miles better than the baseline standard with 95% confidence. Given that the PAT subclass of the trucking industry outperforms the baseline standard at the 95% confidence level, the Company should inquire into the current PAT programs of its insured customers to determine if there exists a root cause that links PAT programs and the reduction in frequency of commercial auto claims. If there appears to be a root cause for this decrease in claims frequency, the Company should incentivize its insured customers to implement this practice.

Additionally, the results in Subsection 5.5 confirmed with 90% confidence that the RTW subclass of the trucking industry demonstrated a worse claims frequency than the baseline performance standard by at least 0.01 claims per million miles. Although unlikely, the Company should scrutinize the return-to-work practices of its insured customers and determine if there is a root cause for this observed worse customer performance amongst the RTW subclass of the trucking industry.

Although the worst-in-class AER implementations and the PAT subclass of the trucking industry were both correlated to some extent with better customer claims frequency performance, the analysis of this thesis ultimately concludes that after analyzing the data provided by the Company, it cannot be shown that the Company’s currently implemented risk management strategies consistently promise significantly improved customer performance with regard to claims frequency or average severity.

Subsection 7.1 will briefly outline the potential for forward collision warning and autonomous emergency braking as new risk management strategies which have the potential to be implemented by the Company.

7.1 Further Applications

An extremely useful further application to this thesis would include fitting a mathematical model to the Company's data with respect to the strategies of forward collision warning and autonomous emergency braking.

According to Cicchino, forward collision warning, autonomous emergency braking, and the combination of the two strategies have the potential to reduce rear-end striking crashes by the following rates:

- FCW: 27%
- AEB: 43%
- FCW with AEB: 50% [1]

The remainder of this section will provide a high-level overview of the potential effectiveness and profitability of these strategies.

In Subsection 4.3, it was reported that claims with the description "Insured rear-ended claimant" constituted 8.4% of the Company's claims. Additionally, claims containing the string "rear" constituted 12.3% of the Company's claims (3,119 of 25,256). In 2014, the Company's customers were involved in 5,149 claims. In 2015, the number of claims dropped to 4,543.

If we assume that the Company's insured customers are responsible for roughly 5,000 claims in a given year, and that 12% of these claims involve rear-end striking crashes, then it can be approximated that the Company's insured customers are involved in roughly 600 claims involving rear-end striking crashes in a given policy year.

The median cost per claim throughout all of the policy years included in the Company's data is \$2,281. For rear-end striking collisions, the median cost per claim rises to \$3,286. If FCW, AEB, or a combination of the two strategies is implemented and performs to the level represented by Cicchino's claims, the Company could roughly expect the following performance improvements from its aggregate book of business for one policy year:

- FCW: 3.2% claims frequency reduction, total incurred cost reduction of \$532,332
- AEB: 5.2% claims frequency reduction, total incurred cost reduction of \$847,788
- FCW with AEB: 6.0% claims frequency reduction, total incurred cost reduction of \$985,800

Given that the Company's data does not necessarily show consistently significant performance improvements with respect to the currently implemented risk management strategies of its insured customers, it would be a reasonable recommendation for the Company to investigate autonomous vehicle technologies and the possible return on investment that these technologies could provide if purposed as risk management strategies.

Appendices

A Appendix: Tables

A.1 Changes in Claims Frequency by AER Year Rank

Table 11: Small Fleet Frequency Changes by Year Rank

Year Rank	YR Frequency	Lag Time Frequency	Percent Change
YR 1	9.57	7.9	-17%
YR 2	9.14	10.25	12%
YR 3	11.68	9.2	-21%

Table 12: Medium-sized Fleet Frequency Changes by Year Rank

Year Rank	YR Frequency	Lag Time Frequency	Percent Change
YR 1	10.58	11.31	7%
YR 2	8.47	13.64	61%
YR 3	11.91	17.22	45%

Table 13: Large Fleet Frequency Changes by Year Rank

Year Rank	YR Frequency	Lag Time Frequency	Percent Change
YR 1	7.18	8.87	24%
YR 2	7.69	8.16	6%
YR 3	7.53	7.18	-5%

A.2 Confidence Intervals for Difference of Means

A.2.1 Defensive Driving Courses and Training

Table 14: Passenger Frequency Confidence Intervals

Confidence Interval	CI Lower Bound	CI Upper Bound
95 Percent	-0.78	2.43
90 Percent	-0.52	2.17
80 Percent	-0.22	1.87

Table 15: Truck Frequency Confidence Intervals

Confidence Interval	CI Lower Bound	CI Upper Bound
95 Percent	-0.14	0.14
90 Percent	-0.12	0.12
80 Percent	-0.10	0.09

Table 16: Passenger Average Severity Confidence Intervals

Confidence Interval	CI Lower Bound	CI Upper Bound
95 Percent	-\$48,865	\$73,793
90 Percent	-\$39,009	\$63,937
80 Percent	-\$27,650	\$52,578

Table 17: Truck Average Severity Confidence Intervals

Confidence Interval	CI Lower Bound	CI Upper Bound
95 Percent	-\$13,956	\$6,985
90 Percent	-\$12,273	\$5,302
80 Percent	-\$10,334	\$3,363

A.2.2 Physical Abilities Testing Programs

Table 18: Passenger Frequency Confidence Intervals

Confidence Interval	CI Lower Bound	CI Upper Bound
95 Percent	-0.83	2.47
90 Percent	-0.57	2.20
80 Percent	-0.26	1.90

Table 19: Truck Frequency Confidence Intervals

Confidence Interval	CI Lower Bound	CI Upper Bound
95 Percent	0.09	0.41
90 Percent	0.12	0.38
80 Percent	0.15	0.35

Table 20: Passenger Average Severity Confidence Intervals

Confidence Interval	CI Lower Bound	CI Upper Bound
95 Percent	-\$46,341	\$70,265
90 Percent	-\$36,971	\$60,895
80 Percent	-\$26,173	\$50,097

Table 21: Truck Average Severity Confidence Intervals

Confidence Interval	CI Lower Bound	CI Upper Bound
95 Percent	-\$20,899	\$7,980
90 Percent	-\$18,579	\$5,659
80 Percent	-\$15,904	\$2,985

A.2.3 Return-to-Work Programs

Table 22: Passenger Frequency Confidence Intervals

Confidence Interval	CI Lower Bound	CI Upper Bound
95 Percent	-1.16	2.66
90 Percent	-0.85	2.36
80 Percent	-0.50	2.00

Table 23: Truck Frequency Confidence Intervals

Confidence Interval	CI Lower Bound	CI Upper Bound
95 Percent	-0.28	0.01
90 Percent	-0.26	-0.01
80 Percent	-0.23	-0.04

Table 24: Passenger Average Severity Confidence Intervals

Confidence Interval	CI Lower Bound	CI Upper Bound
95 Percent	-\$41,668	\$65,131
90 Percent	-\$33,086	\$56,549
80 Percent	-\$23,196	\$46,659

Table 25: Truck Average Severity Confidence Intervals

Confidence Interval	CI Lower Bound	CI Upper Bound
95 Percent	-\$11,172	\$8,399
90 Percent	-\$9,600	\$6,826
80 Percent	-\$7,787	\$5,014

B Appendix: MATLAB Code

B.1 AER.m

```

1 % Tom Byrne
2 % ORFE Senior Thesis
3 % AER.m
4
5 %% Read Data
6
7 % File
8 file = 'CompanyData.xlsx';
9
10 % AER Purchase Order Histories
11 [num,txt,row] = xlsread(file, 'AER');
12
13 %% Overview of AER Purchase Orders
14
15 % Do not include records of expired orders
16
17 trim = num(num(:,2) > 0, 1:9);
18 n = size(trim(:,1));
19
20 % AER Units by Year
21 year = trim(:,8);
22 units_year = zeros(10,1);
23
24 for y = 2006:2015
25     for i = 1:n

```

```

26         if year(i) == y
27             units_year(y-2005) = units_year(y-2005) + trim(i,2);
28         end
29     end
30 end
31
32 % AER Cost by Year
33 cost_year = zeros(10,1);
34
35 for y = 2006:2015
36     for i = 1:n
37         if year(i) == y
38             cost_year(y-2005) = cost_year(y-2005) + trim(i,6);
39         end
40     end
41 end
42
43 % Units and Cost Figure
44
45 figure(1)
46 yyaxis left;
47 bar(2005.79:2014.79,units_year,0.4,'blue');
48 hold on;
49 xticks([2006 2007 2008 2009 2010 2011 2012 2013 2014 2015]);
50 xlabel('Year');
51 ylabel('AER Units Purchased');
52 yyaxis right;
53 bar(2006.21:2015.21,cost_year./10^6,0.4,'red');
54 xticks([2006 2007 2008 2009 2010 2011 2012 2013 2014 2015]);
55 ytickformat('$%,.0f M');
56 xlabel('Year');
57 ylabel('AER Total Cost (USD)');
58 set(gca,'fontsize',20);
59 xlim([2005.25 2015.75]);
60

```

```

61 % Increase in Cost from 2014 to 2015
62 increase = (cost_year(10)-cost_year(9))/cost_year(9);
63 display(increase);
64
65 %% AER Vendor Overview
66
67 vendor = trim(:,9);
68 units_vendor = zeros(6,1);
69
70 for v = 1:6
71     for i = 1:n
72         if vendor(i) == v
73             units_vendor(v) = units_vendor(v) + trim(i,2);
74         end
75     end
76 end
77
78 figure(2)
79 bar(1:6,units_vendor./1000,0.7,'blue');
80 xticks([1 2 3 4 5 6]);
81 ytickformat('%,.0f K');
82 xlabel('Vendor');
83 ylabel('AER Units Purchased');
84 set(gca,'fontsize',20);
85
86 % Table 1: Breakdown of AER Units, by Vendor, by Year
87 vendor_year = zeros(10,6);
88 for v = 1:6
89     for y = 2006:2015
90         for i = 1:n
91             if (vendor(i) == v && year(i) == y)
92                 vendor_year(y-2005,v) = vendor_year(y-2005,v) + trim(i,2);
93             end
94         end
95     end

```

```

96 end
97
98 percentages = vendor_year ./ units_year;
99
100 Vendor1 = percentages(:,1);
101 Vendor2 = percentages(:,2);
102 Vendor3 = percentages(:,3);
103 Vendor4 = percentages(:,4);
104 Vendor5 = percentages(:,5);
105 Vendor6 = percentages(:,6);
106
107 Year = {'2006'; '2007'; '2008'; '2009'; '2010'; ...
108        '2011'; '2012'; '2013'; '2014'; '2015'};
109 T = table(Vendor1, Vendor2, Vendor3, Vendor4, ...
110          Vendor5, Vendor6, 'RowNames', Year);
111
112 writetable(T, 'vendor_year_table.xlsx', 'WriteRowNames', true);

```

B.2 Loss_Analysis.m

```
1 % Tom Byrne
2 % ORFE Senior Thesis
3 % Loss_Analysis.m
4
5 %% Read Data
6
7 % File
8 file = 'CompanyData.xlsx';
9
10 % Losses
11 loss = xlsread(file, 'Losses');
12
13 % Total Incurred Ranges
14 tir = xlsread(file, 'TI Range');
15
16 %% Losses
17
18 % Total Incurred Costs Histogram
19 figure(1)
20 bar(tir,0.7,'blue');
21 names = {'$0-500'; '$500-1k'; '$1k-2k'; '$2k-3k'; ...
22         '$3k-4k'; '$4k-5k'; '$5k-10k'; '$10k+'};
23 set(gca,'xticklabel',names,'fontsize',20);
24 xlabel('Total Incurred Cost of Claims (USD)');
25 ylabel('Frequency (Number of Claims)');
26
27 % Distribution Tail
28 ti = loss(:,8);
29 ti = sort(ti,'descend');
30 ti = ti(ti>500000);
31
32 figure(2)
33 bar(10:length(ti)+9,ti./1000000,'blue');
```

```
34 ytickformat('$.0f M');
35 set(gca,'XTick',[]);
36 axis([1 (length(ti)+20) 0 8]);
37 xlabel('Claims with Total Incurred Cost > $500,000 (n = 153)');
38 ylabel('Total Incurred Cost (USD)');
39 set(gca,'fontsize',20);
40
41 length(ti)
```

B.3 Best_Worst.m

```
1 % Tom Byrne
2 % ORFE Senior Thesis
3 % Best_Worst.m
4
5 %% Read Data
6
7 % File
8 file = 'CompanyData.xlsx';
9
10 % Passenger
11 pass = xlsread(file, 'BWP');
12
13 % Truck
14 truck = xlsread(file, 'BWT');
15
16 %% Passenger Summary
17
18 % Frequency
19 pf_mean = pass(:,1);
20 pf_err = pass(:,2);
21
22 % Total Incurred
23 pti_mean = pass(:,5);
24 pti_err = pass(:,6);
25
26 figure(1)
27 yyaxis left;
28 bar(0.85:3.85, pf_mean, 0.2, 'blue');
29 hold on;
30 errorbar(0.85:3.85, pf_mean, pf_err, 'ko', 'Linewidth', 4);
31 ylabel('Frequency (per 100 Units)');
32 yyaxis right;
33 bar(1.15:4.15, pti_mean./1000, 0.2, 'red');
```

```

34 errorbar(1.15:4.15,pti_mean./1000,pti_err./1000,'ko','Linewidth',4);
35 ytickformat('$%,.0f K');
36 ylabel('Average Severity (per 100 Units)');
37 xlabel('Risk Management Strategy');
38 names = {''; 'AER Best'; ''; 'AER Worst'; ''; ...
39         'All Other AER'; ''; 'Non-AER'; ''};
40 set(gca,'xticklabel',names,'fontsize',20);
41
42 %% Truck Summary
43
44 % Frequency
45 tf_mean = truck(:,1);
46 tf_err = truck(:,2);
47
48 % Total Incurred
49 tti_mean = truck(:,5);
50 tti_err = truck(:,6);
51
52 figure(2)
53 yyaxis left;
54 bar(0.85:3.85,tf_mean,0.2,'blue');
55 hold on;
56 errorbar(0.85:3.85,tf_mean,tf_err,'ko','Linewidth',4);
57 ylabel('Frequency (per 1M miles)');
58 yyaxis right;
59 bar(1.15:4.15,tti_mean./1000,0.2,'red');
60 errorbar(1.15:4.15,tti_mean./1000,tti_err./1000,'ko','Linewidth',4);
61 ytickformat('$%,.0f K');
62 ylabel('Average Severity (per 1M miles)');
63 xlabel('Risk Management Strategy');
64 names = {''; 'AER Best'; ''; 'AER Worst'; ''; ...
65         'All Other AER'; ''; 'Non-AER'; ''};
66 set(gca,'xticklabel',names,'fontsize',20);

```


B.4 Defensive_Driving.m

```
1 % Tom Byrne
2 % ORFE Senior Thesis
3 % Defensive_Driving.m
4
5 name = 'Defensive Driving';
6
7 %% Read Data
8
9 % File
10 file = 'CompanyData.xlsx';
11
12 % Baseline
13 base = xlsread(file, 'Baseline');
14
15 % Defensive Driving Courses
16 strat = xlsread(file, 'DD');
17
18 % Row Indices of Data
19 row_p = 11:20;
20 row_t = 11:16;
21
22 %% Number of Data Points
23
24 np = strat(row_p,11);
25 np = num2str(np);
26 np = cellstr(np);
27 for i = 1:10
28     np{i} = strcat('n = ',np{i});
29 end
30
31 nt = strat(row_t,15);
32 nt = num2str(nt);
33 nt = cellstr(nt);
```

```

34 for i = 1:6
35     nt{i} = strcat('n = ',nt{i});
36 end
37
38 %% Calculate Frequencies
39
40 % Passenger Frequency
41 claims_p = strat(row_p,2);
42 units = strat(row_p,3);
43 freq_p = (claims_p.*100)./units;
44 err_p = strat(row_p,4);
45
46 % Truck Frequency
47 claims_t = strat(row_t,6);
48 miles = strat(row_t,7);
49 freq_t = (claims_t.*1000000)./miles;
50 err_t = strat(row_t,8);
51
52 % Baseline: Passenger Frequency
53 claims_p = base(row_p,2);
54 units = base(row_p,3);
55 base_p = (claims_p.*100)./units;
56 err_b_p = base(row_p,4);
57
58 % Baseline: Truck Frequency
59 claims_t = base(row_t,6);
60 miles = base(row_t,7);
61 base_t = (claims_t.*1000000)./miles;
62 err_b_t = base(row_t,8);
63
64 %% Calculate Average Severity
65
66 % Passenger Average Severity
67 ti_p = strat(row_p,10);
68 units = strat(row_p,3);

```

```

69 avgti_p = (ti_p.*100)./units;
70 err_p_ti = strat(row_p,12);
71
72 % Truck Average Severity
73 ti_t = strat(row_t,14);
74 miles = strat(row_t,7);
75 avgti_t = (ti_t.*1000000)./miles;
76 err_t_ti = strat(row_t,16);
77
78 % Baseline: Passenger Average Severity
79 ti_p = base(row_p,10);
80 units = base(row_p,3);
81 avgti_p_base = (ti_p.*100)./units;
82 err_p_ti_base = base(row_p,12);
83
84 % Baseline: Truck Average Severity
85 ti_t = base(row_t,14);
86 miles = base(row_t,7);
87 avgti_t_base = (ti_t.*1000000)./miles;
88 err_t_ti_base = base(row_t,16);
89
90 %% Passenger Figure
91
92 % Passenger vs. Baseline: Frequency
93 figure(1)
94 subplot(2,1,1);
95 errorbar(2005.925:2014.925,base_p, err_b_p, '—r', 'LineWidth',1);
96 hold on
97 errorbar(2006.075:2015.075,freq_p, err_p, '-bo', 'LineWidth',2,...
98     'MarkerSize',8, 'MarkerFaceColor','blue');
99 legend('Baseline Passenger',strcat(name,' Passenger'));
100 set(gca, 'fontsize',14);
101 ylabel('Frequency (per 100 Units)');
102 axis([2005.5 2015.5 0 12]);
103 a = ylim;

```

```

104 text(2005.9:2014.9,a(2).*1.05.*ones(10,1),np,'FontSize',10);
105
106 % Passenger vs. Baseline: Average Severity
107 subplot(2,1,2);
108 errorbar(2005.925:2014.925,avgti_p_base./1000,...
109     err_p_ti_base./1000,'-r','LineWidth',1);
110 hold on
111 errorbar(2006.075:2015.075,avgti_p./1000,err_p_ti./1000,'-bo',...
112     'LineWidth',2,'MarkerSize',8,'MarkerFaceColor','blue');
113 set(gca,'fontsize',14);
114 xlabel('Policy Effective Year');
115 ylabel('Average Severity (per 100 Units)');
116 ytickformat('$%,.0f K');
117 axis([2005.5 2015.5 0 250]);
118 a = ylim;
119 text(2005.9:2014.9,a(2).*1.05.*ones(10,1),np,'FontSize',10);
120
121 %% Truck Figure
122
123 % Truck vs. Baseline: Frequency
124 figure(2)
125 subplot(2,1,1);
126 errorbar(2009.95:2014.95,base_t,err_b_t,'-r','LineWidth',1);
127 hold on
128 errorbar(2010.05:2015.05,freq_t,err_t,'-bo','LineWidth',2,...
129     'MarkerSize',8,'MarkerFaceColor','blue');
130 legend('Baseline Truck',strcat(name,' Truck'));
131 set(gca,'fontsize',14);
132 ylabel('Frequency (per 1M miles)');
133 axis([2009.5 2015.5 0.5 2.5]);
134 a = ylim;
135 text(2009.9:2014.9,a(2).*1.05.*ones(6,1),nt,'FontSize',10);
136
137 % Truck vs. Baseline: Average Severity
138 subplot(2,1,2);

```

```

139 errorbar(2009.95:2014.95,avgti_t_base./1000,...
140         err_t_ti_base./1000,'-r','LineWidth',1);
141 hold on
142 errorbar(2010.05:2015.05,avgti_t./1000,err_t_ti./1000,'-bo',...
143         'LineWidth',2,'MarkerSize',8,'MarkerFaceColor','blue');
144 set(gca,'fontsize',14);
145 xlabel('Policy Effective Year');
146 ylabel('Average Severity (per 1M miles)');
147 ytickformat('$%,.0f K');
148 axis([2009.5 2015.5 0 80]);
149 a = ylim;
150 text(2009.9:2014.9,a(2).*1.05.*ones(6,1),nt,'FontSize',10);
151
152 %% Confidence Interval Variables
153
154 % Baseline
155 bpf_v = base(5,4);
156 bpf_n = base(6,4);
157 bpf_m = base(5,3);
158 btf_v = base(5,8);
159 btf_n = base(6,8);
160 btf_m = base(5,7);
161 bpt_v = base(5,12);
162 bpt_n = base(6,12);
163 bpt_m = base(5,11);
164 btt_v = base(5,16);
165 btt_n = base(6,16);
166 btt_m = base(5,15);
167
168 % Strategy
169 spf_v = strat(5,4);
170 spf_n = strat(6,4);
171 spf_m = strat(5,3);
172 stf_v = strat(5,8);
173 stf_n = strat(6,8);

```

```

174 stf_m = strat(5,7);
175 spt_v = strat(5,12);
176 spt_n = strat(6,12);
177 spt_m = strat(5,11);
178 stt_v = strat(5,16);
179 stt_n = strat(6,16);
180 stt_m = strat(5,15);
181
182 %% 95 Percent Confidence Intervals
183 t = 1.96;
184
185 % Passenger Frequency
186 dat(1,1) = bpf_m - spf_m - t*sqrt((bpf_v/bpf_n)+(spf_v/spf_n));
187 dat(1,2) = bpf_m - spf_m + t*sqrt((bpf_v/bpf_n)+(spf_v/spf_n));
188
189 % Truck Frequency
190 dat(2,1) = btf_m - stf_m - t*sqrt((btf_v/btf_n)+(stf_v/stf_n));
191 dat(2,2) = btf_m - stf_m + t*sqrt((btf_v/btf_n)+(stf_v/stf_n));
192
193 % Passenger Average Severity
194 dat(3,1) = bpt_m - spt_m - t*sqrt((bpt_v/bpt_n)+(spt_v/spt_n));
195 dat(3,2) = bpt_m - spt_m + t*sqrt((bpt_v/bpt_n)+(spt_v/spt_n));
196
197 % Truck Average Severity
198 dat(4,1) = btt_m - stt_m - t*sqrt((btt_v/btt_n)+(stt_v/stt_n));
199 dat(4,2) = btt_m - stt_m + t*sqrt((btt_v/btt_n)+(stt_v/stt_n));
200
201 %% 90 Percent Confidence Intervals
202 t = 1.645;
203
204 % Passenger Frequency
205 dat(6,1) = bpf_m - spf_m - t*sqrt((bpf_v/bpf_n)+(spf_v/spf_n));
206 dat(6,2) = bpf_m - spf_m + t*sqrt((bpf_v/bpf_n)+(spf_v/spf_n));
207
208 % Truck Frequency

```

```

209 dat(7,1) = btf_m - stf_m - t*sqrt((btf_v/btf_n)+(stf_v/stf_n));
210 dat(7,2) = btf_m - stf_m + t*sqrt((btf_v/btf_n)+(stf_v/stf_n));
211
212 % Passenger Average Severity
213 dat(8,1) = bpt_m - spt_m - t*sqrt((bpt_v/bpt_n)+(spt_v/spt_n));
214 dat(8,2) = bpt_m - spt_m + t*sqrt((bpt_v/bpt_n)+(spt_v/spt_n));
215
216 % Truck Average Severity
217 dat(9,1) = btt_m - stt_m - t*sqrt((btt_v/btt_n)+(stt_v/stt_n));
218 dat(9,2) = btt_m - stt_m + t*sqrt((btt_v/btt_n)+(stt_v/stt_n));
219
220 %% 80 Percent Confidence Intervals
221 t = 1.282;
222
223 % Passenger Frequency
224 dat(11,1) = bpf_m - spf_m - t*sqrt((bpf_v/bpf_n)+(spf_v/spf_n));
225 dat(11,2) = bpf_m - spf_m + t*sqrt((bpf_v/bpf_n)+(spf_v/spf_n));
226
227 % Truck Frequency
228 dat(12,1) = btf_m - stf_m - t*sqrt((btf_v/btf_n)+(stf_v/stf_n));
229 dat(12,2) = btf_m - stf_m + t*sqrt((btf_v/btf_n)+(stf_v/stf_n));
230
231 % Passenger Average Severity
232 dat(13,1) = bpt_m - spt_m - t*sqrt((bpt_v/bpt_n)+(spt_v/spt_n));
233 dat(13,2) = bpt_m - spt_m + t*sqrt((bpt_v/bpt_n)+(spt_v/spt_n));
234
235 % Truck Average Severity
236 dat(14,1) = btt_m - stt_m - t*sqrt((btt_v/btt_n)+(stt_v/stt_n));
237 dat(14,2) = btt_m - stt_m + t*sqrt((btt_v/btt_n)+(stt_v/stt_n));
238
239 %% Write Data
240 csvwrite(strcat(name,' data.csv'),dat);

```

B.5 PAT.m

Note: The code for **PAT.m** is nearly identical to that of **Defensive.Driving.m**, with the exception of commands which read in the data and set figure axis limits.

```
1 % Tom Byrne
2 % ORFE Senior Thesis
3 % PAT.m
4
5 name = 'PAT';
6
7 %% Read Data
8
9 % File
10 file = 'CompanyData.xlsx';
11
12 % Baseline
13 base = xlsread(file, 'Baseline');
14
15 % Physical Abilities Testing (abbr. PAT) Programs
16 strat = xlsread(file, 'PAT');
17
18 % Row Indices of Data
19 row_p = 11:20;
20 row_t = 11:16;
21
22 %% Number of Data Points
23
24 np = strat(row_p,11);
25 np = num2str(np);
26 np = cellstr(np);
27 for i = 1:10
28     np{i} = strcat('n = ',np{i});
29 end
30
31 nt = strat(row_t,15);
```



```

32 nt = num2str(nt);
33 nt = cellstr(nt);
34 for i = 1:6
35     nt{i} = strcat('n = ',nt{i});
36 end
37
38 %% Calculate Frequencies
39
40 % Passenger Frequency
41 claims_p = strat(row_p,2);
42 units = strat(row_p,3);
43 freq_p = (claims_p.*100)./units;
44 err_p = strat(row_p,4);
45
46 % Truck Frequency
47 claims_t = strat(row_t,6);
48 miles = strat(row_t,7);
49 freq_t = (claims_t.*1000000)./miles;
50 err_t = strat(row_t,8);
51
52 % Baseline: Passenger Frequency
53 claims_p = base(row_p,2);
54 units = base(row_p,3);
55 base_p = (claims_p.*100)./units;
56 err_b_p = base(row_p,4);
57
58 % Baseline: Truck Frequency
59 claims_t = base(row_t,6);
60 miles = base(row_t,7);
61 base_t = (claims_t.*1000000)./miles;
62 err_b_t = base(row_t,8);
63
64 %% Calculate Average Severity
65
66 % Passenger Average Severity

```

```

67 ti_p = strat(row_p,10);
68 units = strat(row_p,3);
69 avgti_p = (ti_p.*100)./units;
70 err_p_ti = strat(row_p,12);
71
72 % Truck Average Severity
73 ti_t = strat(row_t,14);
74 miles = strat(row_t,7);
75 avgti_t = (ti_t.*1000000)./miles;
76 err_t_ti = strat(row_t,16);
77
78 % Baseline: Passenger Average Severity
79 ti_p = base(row_p,10);
80 units = base(row_p,3);
81 avgti_p_base = (ti_p.*100)./units;
82 err_p_ti_base = base(row_p,12);
83
84 % Baseline: Truck Average Severity
85 ti_t = base(row_t,14);
86 miles = base(row_t,7);
87 avgti_t_base = (ti_t.*1000000)./miles;
88 err_t_ti_base = base(row_t,16);
89
90 %% Passenger Figure
91
92 % Passenger vs. Baseline: Frequency
93 figure(1)
94 subplot(2,1,1);
95 errorbar(2005.925:2014.925,base_p,err_b_p,'-r','LineWidth',1);
96 hold on
97 errorbar(2006.075:2015.075,freq_p,err_p,'-bo','LineWidth',2,...
98     'MarkerSize',8,'MarkerFaceColor','blue');
99 legend('Baseline Passenger',strcat(name,' Passenger'));
100 set(gca,'fontSize',14);
101 ylabel('Frequency (per 100 Units)');

```

```

102 axis([2005.5 2015.5 0 12]);
103 a = ylim;
104 text(2005.9:2014.9,a(2).*1.05.*ones(10,1),np,'FontSize',10);
105
106 % Passenger vs. Baseline: Average Severity
107 subplot(2,1,2);
108 errorbar(2005.925:2014.925,avgti_p_base./1000,...
109         err_p_ti_base./1000,'-r','LineWidth',1);
110 hold on
111 errorbar(2006.075:2015.075,avgti_p./1000,err_p_ti./1000,'-bo',...
112         'LineWidth',2,'MarkerSize',8,'MarkerFaceColor','blue');
113 set(gca,'fontsize',14);
114 xlabel('Policy Effective Year');
115 ylabel('Average Severity (per 100 Units)');
116 ytickformat('$%,0f K');
117 axis([2005.5 2015.5 0 250]);
118 a = ylim;
119 text(2005.9:2014.9,a(2).*1.05.*ones(10,1),np,'FontSize',10);
120
121 %% Truck Figure
122
123 % Truck vs. Baseline: Frequency
124 figure(2)
125 subplot(2,1,1);
126 errorbar(2009.95:2014.95,base_t,err_b_t,'-r','LineWidth',1);
127 hold on
128 errorbar(2010.05:2015.05,freq_t,err_t,'-bo','LineWidth',2,...
129         'MarkerSize',8,'MarkerFaceColor','blue');
130 legend('Baseline Truck',strcat(name,' Truck'));
131 set(gca,'fontsize',14);
132 ylabel('Frequency (per 1M miles)');
133 axis([2009.5 2015.5 0 2]);
134 a = ylim;
135 text(2009.9:2014.9,a(2).*1.05.*ones(6,1),nt,'FontSize',10);
136

```

```

137 % Truck vs. Baseline: Average Severity
138 subplot(2,1,2);
139 errorbar(2009.95:2014.95,avgti_t_base./1000,...
140         err_t_ti_base./1000,'—r','LineWidth',1);
141 hold on
142 errorbar(2010.05:2015.05,avgti_t./1000,err_t_ti./1000,'-bo',...
143         'LineWidth',2,'MarkerSize',8,'MarkerFaceColor','blue');
144 set(gca,'fontsize',14);
145 xlabel('Policy Effective Year');
146 ylabel('Average Severity (per 1M miles)');
147 ytickformat('$%,.0f K');
148 axis([2009.5 2015.5 0 200]);
149 a = ylim;
150 text(2009.9:2014.9,a(2).*1.05.*ones(6,1),nt,'FontSize',10);
151
152 %% Confidence Interval Variables
153
154 % Baseline
155 bpf_v = base(5,4);
156 bpf_n = base(6,4);
157 bpf_m = base(5,3);
158 btf_v = base(5,8);
159 btf_n = base(6,8);
160 btf_m = base(5,7);
161 bpt_v = base(5,12);
162 bpt_n = base(6,12);
163 bpt_m = base(5,11);
164 btt_v = base(5,16);
165 btt_n = base(6,16);
166 btt_m = base(5,15);
167
168 % Strategy
169 spf_v = strat(5,4);
170 spf_n = strat(6,4);
171 spf_m = strat(5,3);

```

```

172 stf_v = strat(5,8);
173 stf_n = strat(6,8);
174 stf_m = strat(5,7);
175 spt_v = strat(5,12);
176 spt_n = strat(6,12);
177 spt_m = strat(5,11);
178 stt_v = strat(5,16);
179 stt_n = strat(6,16);
180 stt_m = strat(5,15);
181
182 %% 95 Percent Confidence Intervals
183 t = 1.96;
184
185 % Passenger Frequency
186 dat(1,1) = bpf_m - spf_m - t*sqrt((bpf_v/bpf_n)+(spf_v/spf_n));
187 dat(1,2) = bpf_m - spf_m + t*sqrt((bpf_v/bpf_n)+(spf_v/spf_n));
188
189 % Truck Frequency
190 dat(2,1) = btf_m - stf_m - t*sqrt((btf_v/btf_n)+(stf_v/stf_n));
191 dat(2,2) = btf_m - stf_m + t*sqrt((btf_v/btf_n)+(stf_v/stf_n));
192
193 % Passenger Average Severity
194 dat(3,1) = bpt_m - spt_m - t*sqrt((bpt_v/bpt_n)+(spt_v/spt_n));
195 dat(3,2) = bpt_m - spt_m + t*sqrt((bpt_v/bpt_n)+(spt_v/spt_n));
196
197 % Truck Average Severity
198 dat(4,1) = btt_m - stt_m - t*sqrt((btt_v/btt_n)+(stt_v/stt_n));
199 dat(4,2) = btt_m - stt_m + t*sqrt((btt_v/btt_n)+(stt_v/stt_n));
200
201 %% 90 Percent Confidence Intervals
202 t = 1.645;
203
204 % Passenger Frequency
205 dat(6,1) = bpf_m - spf_m - t*sqrt((bpf_v/bpf_n)+(spf_v/spf_n));
206 dat(6,2) = bpf_m - spf_m + t*sqrt((bpf_v/bpf_n)+(spf_v/spf_n));

```

```

207
208 % Truck Frequency
209 dat(7,1) = btf_m - stf_m - t*sqrt((btf_v/btf_n)+(stf_v/stf_n));
210 dat(7,2) = btf_m - stf_m + t*sqrt((btf_v/btf_n)+(stf_v/stf_n));
211
212 % Passenger Average Severity
213 dat(8,1) = bpt_m - spt_m - t*sqrt((bpt_v/bpt_n)+(spt_v/spt_n));
214 dat(8,2) = bpt_m - spt_m + t*sqrt((bpt_v/bpt_n)+(spt_v/spt_n));
215
216 % Truck Average Severity
217 dat(9,1) = btt_m - stt_m - t*sqrt((btt_v/btt_n)+(stt_v/stt_n));
218 dat(9,2) = btt_m - stt_m + t*sqrt((btt_v/btt_n)+(stt_v/stt_n));
219
220 %% 80 Percent Confidence Intervals
221 t = 1.282;
222
223 % Passenger Frequency
224 dat(11,1) = bpf_m - spf_m - t*sqrt((bpf_v/bpf_n)+(spf_v/spf_n));
225 dat(11,2) = bpf_m - spf_m + t*sqrt((bpf_v/bpf_n)+(spf_v/spf_n));
226
227 % Truck Frequency
228 dat(12,1) = btf_m - stf_m - t*sqrt((btf_v/btf_n)+(stf_v/stf_n));
229 dat(12,2) = btf_m - stf_m + t*sqrt((btf_v/btf_n)+(stf_v/stf_n));
230
231 % Passenger Average Severity
232 dat(13,1) = bpt_m - spt_m - t*sqrt((bpt_v/bpt_n)+(spt_v/spt_n));
233 dat(13,2) = bpt_m - spt_m + t*sqrt((bpt_v/bpt_n)+(spt_v/spt_n));
234
235 % Truck Average Severity
236 dat(14,1) = btt_m - stt_m - t*sqrt((btt_v/btt_n)+(stt_v/stt_n));
237 dat(14,2) = btt_m - stt_m + t*sqrt((btt_v/btt_n)+(stt_v/stt_n));
238
239 %% Write Data
240 csvwrite(strcat(name, ' data.csv'), dat);

```

B.6 RTW.m

Note: The code for **RTW.m** is nearly identical to that of **Defensive_Driving.m**, with the exception of commands which read in the data and set figure axis limits.

```
1 % Tom Byrne
2 % ORFE Senior Thesis
3 % RTW.m
4
5 name = 'RIW';
6
7 %% Read Data
8
9 % File
10 file = 'CompanyData.xlsx';
11
12 % Baseline
13 base = xlsread(file, 'Baseline');
14
15 % Return-to-Work (abbr. RIW) Programs
16 strat = xlsread(file, 'RIW');
17
18 % Row Indices of Data
19 row_p = 11:20;
20 row_t = 11:16;
21
22 %% Number of Data Points
23
24 np = strat(row_p,11);
25 np = num2str(np);
26 np = cellstr(np);
27 for i = 1:10
28     np{i} = strcat('n = ',np{i});
29 end
30
31 nt = strat(row_t,15);
```

```

32 nt = num2str(nt);
33 nt = cellstr(nt);
34 for i = 1:6
35     nt{i} = strcat('n = ',nt{i});
36 end
37
38 %% Calculate Frequencies
39
40 % Passenger Frequency
41 claims_p = strat(row_p,2);
42 units = strat(row_p,3);
43 freq_p = (claims_p.*100)./units;
44 err_p = strat(row_p,4);
45
46 % Truck Frequency
47 claims_t = strat(row_t,6);
48 miles = strat(row_t,7);
49 freq_t = (claims_t.*1000000)./miles;
50 err_t = strat(row_t,8);
51
52 % Baseline: Passenger Frequency
53 claims_p = base(row_p,2);
54 units = base(row_p,3);
55 base_p = (claims_p.*100)./units;
56 err_b_p = base(row_p,4);
57
58 % Baseline: Truck Frequency
59 claims_t = base(row_t,6);
60 miles = base(row_t,7);
61 base_t = (claims_t.*1000000)./miles;
62 err_b_t = base(row_t,8);
63
64 %% Calculate Average Severity
65
66 % Passenger Average Severity

```



```

67 ti_p = strat(row_p,10);
68 units = strat(row_p,3);
69 avgti_p = (ti_p.*100)./units;
70 err_p_ti = strat(row_p,12);
71
72 % Truck Average Severity
73 ti_t = strat(row_t,14);
74 miles = strat(row_t,7);
75 avgti_t = (ti_t.*1000000)./miles;
76 err_t_ti = strat(row_t,16);
77
78 % Baseline: Passenger Average Severity
79 ti_p = base(row_p,10);
80 units = base(row_p,3);
81 avgti_p_base = (ti_p.*100)./units;
82 err_p_ti_base = base(row_p,12);
83
84 % Baseline: Truck Average Severity
85 ti_t = base(row_t,14);
86 miles = base(row_t,7);
87 avgti_t_base = (ti_t.*1000000)./miles;
88 err_t_ti_base = base(row_t,16);
89
90 %% Passenger Figure
91
92 % Passenger vs. Baseline: Frequency
93 figure(1)
94 subplot(2,1,1);
95 errorbar(2005.925:2014.925,base_p,err_b_p,'-r','LineWidth',1);
96 hold on
97 errorbar(2006.075:2015.075,freq_p,err_p,'-bo','LineWidth',2,...
98     'MarkerSize',8,'MarkerFaceColor','blue');
99 legend('Baseline Passenger',strcat(name,' Passenger'));
100 set(gca,'fontSize',14);
101 ylabel('Frequency (per 100 Units)');

```

```

102 axis([2005.5 2015.5 0 12]);
103 a = ylim;
104 text(2005.9:2014.9,a(2).*1.05.*ones(10,1),np,'FontSize',10);
105
106 % Passenger vs. Baseline: Average Severity
107 subplot(2,1,2);
108 errorbar(2005.925:2014.925,avgti_p_base./1000,...
109         err_p_ti_base./1000,'-r','LineWidth',1);
110 hold on
111 errorbar(2006.075:2015.075,avgti_p./1000,err_p_ti./1000,'-bo',...
112         'LineWidth',2,'MarkerSize',8,'MarkerFaceColor','blue');
113 set(gca,'fontsize',14);
114 xlabel('Policy Effective Year');
115 ylabel('Average Severity (per 100 Units)');
116 ytickformat('$%,0f K');
117 axis([2005.5 2015.5 0 250]);
118 a = ylim;
119 text(2005.9:2014.9,a(2).*1.05.*ones(10,1),np,'FontSize',10);
120
121 %% Truck Figure
122
123 % Truck vs. Baseline: Frequency
124 figure(2)
125 subplot(2,1,1);
126 errorbar(2009.95:2014.95,base_t,err_b_t,'-r','LineWidth',1);
127 hold on
128 errorbar(2010.05:2015.05,freq_t,err_t,'-bo','LineWidth',2,...
129         'MarkerSize',8,'MarkerFaceColor','blue');
130 legend('Baseline Truck',strcat(name,' Truck'));
131 set(gca,'fontsize',14);
132 ylabel('Frequency (per 1M miles)');
133 axis([2009.5 2015.5 0.5 2.5]);
134 a = ylim;
135 text(2009.9:2014.9,a(2).*1.05.*ones(6,1),nt,'FontSize',10);
136

```

```

137 % Truck vs. Baseline: Average Severity
138 subplot(2,1,2);
139 errorbar(2009.95:2014.95,avgti_t_base./1000,...
140         err_t_ti_base./1000,'—r','LineWidth',1);
141 hold on
142 errorbar(2010.05:2015.05,avgti_t./1000,err_t_ti./1000,'-bo',...
143         'LineWidth',2,'MarkerSize',8,'MarkerFaceColor','blue');
144 set(gca,'fontsize',14);
145 xlabel('Policy Effective Year');
146 ylabel('Average Severity (per 1M miles)');
147 ytickformat('$%,.0f K');
148 axis([2009.5 2015.5 0 100]);
149 a = ylim;
150 text(2009.9:2014.9,a(2).*1.05.*ones(6,1),nt,'FontSize',10);
151
152 %% Confidence Interval Variables
153
154 % Baseline
155 bpf_v = base(5,4);
156 bpf_n = base(6,4);
157 bpf_m = base(5,3);
158 btf_v = base(5,8);
159 btf_n = base(6,8);
160 btf_m = base(5,7);
161 bpt_v = base(5,12);
162 bpt_n = base(6,12);
163 bpt_m = base(5,11);
164 btt_v = base(5,16);
165 btt_n = base(6,16);
166 btt_m = base(5,15);
167
168 % Strategy
169 spf_v = strat(5,4);
170 spf_n = strat(6,4);
171 spf_m = strat(5,3);

```

```

172 stf_v = strat(5,8);
173 stf_n = strat(6,8);
174 stf_m = strat(5,7);
175 spt_v = strat(5,12);
176 spt_n = strat(6,12);
177 spt_m = strat(5,11);
178 stt_v = strat(5,16);
179 stt_n = strat(6,16);
180 stt_m = strat(5,15);
181
182 %% 95 Percent Confidence Intervals
183 t = 1.96;
184
185 % Passenger Frequency
186 dat(1,1) = bpf_m - spf_m - t*sqrt((bpf_v/bpf_n)+(spf_v/spf_n));
187 dat(1,2) = bpf_m - spf_m + t*sqrt((bpf_v/bpf_n)+(spf_v/spf_n));
188
189 % Truck Frequency
190 dat(2,1) = btf_m - stf_m - t*sqrt((btf_v/btf_n)+(stf_v/stf_n));
191 dat(2,2) = btf_m - stf_m + t*sqrt((btf_v/btf_n)+(stf_v/stf_n));
192
193 % Passenger Average Severity
194 dat(3,1) = bpt_m - spt_m - t*sqrt((bpt_v/bpt_n)+(spt_v/spt_n));
195 dat(3,2) = bpt_m - spt_m + t*sqrt((bpt_v/bpt_n)+(spt_v/spt_n));
196
197 % Truck Average Severity
198 dat(4,1) = btt_m - stt_m - t*sqrt((btt_v/btt_n)+(stt_v/stt_n));
199 dat(4,2) = btt_m - stt_m + t*sqrt((btt_v/btt_n)+(stt_v/stt_n));
200
201 %% 90 Percent Confidence Intervals
202 t = 1.645;
203
204 % Passenger Frequency
205 dat(6,1) = bpf_m - spf_m - t*sqrt((bpf_v/bpf_n)+(spf_v/spf_n));
206 dat(6,2) = bpf_m - spf_m + t*sqrt((bpf_v/bpf_n)+(spf_v/spf_n));

```

```

207
208 % Truck Frequency
209 dat(7,1) = btf_m - stf_m - t*sqrt((btf_v/btf_n)+(stf_v/stf_n));
210 dat(7,2) = btf_m - stf_m + t*sqrt((btf_v/btf_n)+(stf_v/stf_n));
211
212 % Passenger Average Severity
213 dat(8,1) = bpt_m - spt_m - t*sqrt((bpt_v/bpt_n)+(spt_v/spt_n));
214 dat(8,2) = bpt_m - spt_m + t*sqrt((bpt_v/bpt_n)+(spt_v/spt_n));
215
216 % Truck Average Severity
217 dat(9,1) = btt_m - stt_m - t*sqrt((btt_v/btt_n)+(stt_v/stt_n));
218 dat(9,2) = btt_m - stt_m + t*sqrt((btt_v/btt_n)+(stt_v/stt_n));
219
220 %% 80 Percent Confidence Intervals
221 t = 1.282;
222
223 % Passenger Frequency
224 dat(11,1) = bpf_m - spf_m - t*sqrt((bpf_v/bpf_n)+(spf_v/spf_n));
225 dat(11,2) = bpf_m - spf_m + t*sqrt((bpf_v/bpf_n)+(spf_v/spf_n));
226
227 % Truck Frequency
228 dat(12,1) = btf_m - stf_m - t*sqrt((btf_v/btf_n)+(stf_v/stf_n));
229 dat(12,2) = btf_m - stf_m + t*sqrt((btf_v/btf_n)+(stf_v/stf_n));
230
231 % Passenger Average Severity
232 dat(13,1) = bpt_m - spt_m - t*sqrt((bpt_v/bpt_n)+(spt_v/spt_n));
233 dat(13,2) = bpt_m - spt_m + t*sqrt((bpt_v/bpt_n)+(spt_v/spt_n));
234
235 % Truck Average Severity
236 dat(14,1) = btt_m - stt_m - t*sqrt((btt_v/btt_n)+(stt_v/stt_n));
237 dat(14,2) = btt_m - stt_m + t*sqrt((btt_v/btt_n)+(stt_v/stt_n));
238
239 %% Write Data
240 csvwrite(strcat(name, ' data.csv'), dat);

```

B.7 Summary.m

```
1 % Tom Byrne
2 % ORFE Senior Thesis
3 % Summary.m
4
5 %% Read Data
6
7 % File
8 file = 'CompanyData.xlsx';
9
10 % Baseline
11 base = xlsread(file, 'Baseline');
12
13 % Defensive Driving Courses
14 def = xlsread(file, 'DD');
15
16 % Physical Abilities Testing (abbr. PAT) Programs
17 pat = xlsread(file, 'PAT');
18
19 % Return-to-Work (abbr. RTW) Programs
20 rtw = xlsread(file, 'RIW');
21
22 %% Passenger Summary
23
24 % Frequency
25
26 base_p = base(5,3);
27 base_p_err = base(6,3);
28
29 def_p = def(5,3);
30 def_p_err = def(6,3);
31
32 pat_p = pat(5,3);
33 pat_p_err = pat(6,3);
```

```

34
35 rtw_p = rtw(5,3);
36 rtw_p_err = rtw(6,3);
37
38 means_p = [base_p; def_p; pat_p; rtw_p];
39 err_p = [base_p_err; def_p_err; pat_p_err; rtw_p_err];
40
41 % Average Severity
42
43 ti_base_p = base(5,11);
44 ti_base_p_err = base(6,11);
45
46 ti_def_p = def(5,11);
47 ti_def_p_err = def(6,11);
48
49 ti_pat_p = pat(5,11);
50 ti_pat_p_err = pat(6,11);
51
52 ti_rtw_p = rtw(5,11);
53 ti_rtw_p_err = rtw(6,11);
54
55 ti_means_p = [ti_base_p; ti_def_p; ti_pat_p; ti_rtw_p];
56 ti_err_p = [ti_base_p_err; ti_def_p_err; ti_pat_p_err; ti_rtw_p_err];
57
58 figure(1)
59 yyaxis left;
60 bar(0.85:3.85, means_p, 0.2, 'blue');
61 hold on;
62 errorbar(0.85:3.85, means_p, err_p, 'ko', 'Linewidth', 4);
63 ylabel('Frequency (per 100 Units)');
64 yyaxis right;
65 bar(1.15:4.15, ti_means_p./1000, 0.2, 'red');
66 errorbar(1.15:4.15, ti_means_p./1000, ti_err_p./1000, 'ko', 'Linewidth', 4);
67 ytickformat('$.0f K');
68 ylabel('Average Severity (per 100 Units)');

```

```

69 xlabel('Risk Management Strategy');
70 names = {''; 'Baseline'; ''; 'Defensive Driving'; ''; ...
71         'PAT'; ''; 'RIW'; ''};
72 set(gca, 'xticklabel', names, 'fontsize', 20);
73
74 %% Truck Summary
75
76 % Frequency
77
78 base_t = base(5,7);
79 base_t_err = base(6,7);
80
81 def_t = def(5,7);
82 def_t_err = def(6,7);
83
84 pat_t = pat(5,7);
85 pat_t_err = pat(6,7);
86
87 rtw_t = rtw(5,7);
88 rtw_t_err = rtw(6,7);
89
90 means_t = [base_t; def_t; pat_t; rtw_t];
91 err_t = [base_t_err; def_t_err; pat_t_err; rtw_t_err];
92
93 % Average Severity
94
95 ti_base_t = base(5,15);
96 ti_base_t_err = base(6,15);
97
98 ti_def_t = def(5,15);
99 ti_def_t_err = def(6,15);
100
101 ti_pat_t = pat(5,15);
102 ti_pat_t_err = pat(6,15);
103

```



```

104 ti_rtw_t = rtw(5,15);
105 ti_rtw_t_err = rtw(6,15);
106
107 ti_means_t = [ti_base_t; ti_def_t; ti_pat_t; ti_rtw_t];
108 ti_err_t = [ti_base_t_err; ti_def_t_err; ti_pat_t_err; ti_rtw_t_err];
109
110 figure(2)
111 yyaxis left;
112 bar(0.85:3.85, means_t, 0.2, 'blue');
113 hold on;
114 errorbar(0.85:3.85, means_t, err_t, 'ko', 'Linewidth', 4);
115 ylabel('Frequency (per 1M miles)');
116 yyaxis right;
117 bar(1.15:4.15, ti_means_t./1000, 0.2, 'red');
118 errorbar(1.15:4.15, ti_means_t./1000, ti_err_t./1000, 'ko', 'Linewidth', 4);
119 ytickformat('$%,.0f K');
120 ylabel('Average Severity (per 1M miles)');
121 xlabel('Risk Management Strategy');
122 names = {''; 'Baseline'; ''; 'Defensive Driving'; ''; ...
123         'PAT'; ''; 'RIW'; ''};
124 set(gca, 'xticklabel', names, 'fontsize', 20);

```

References

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- [3] U.S. Census Bureau (2011). *Population Distribution and Change: 2000 to 2010*. Retrieved from <https://www.census.gov/prod/cen2010/briefs/c2010br-01.pdf>