A filtering approach to tracking volatility from prices observed at random times

Boris Rozovsky
Brown University

This paper is concerned with nonlinear filtering of the coefficients in asset price models with stochastic volatility. More specifically, we assume that the asset price process $S = (S_t)_{t \geq 0}$ is given by

$$dS_t = r(\theta_t) S_t dt + \sqrt{v(\theta_t)} S_t dB_t,$$

where $B = (B_t)_{t \geq 0}$ is a Brownian motion, v is a positive function, and the volatility $\theta = (\theta_t)_{t \geq 0}$ is a a Markov process. The random process θ is unobservable. We assume also that the asset price S_t is observed only discretely, at random times $0 < \tau_1 < \tau_2 < \dots$. This is an appropriate assumption when modeling high frequency financial data (e.g., tick-by-tick stock prices).

In the above setting the problem of estimation of θ can be approached as a special nonlinear filtering problem. While quite natural, this problem does not fit into the "standard" filtering framework and requires new technical tools. We derive the optimal recursive Bayesian filter for θ_t in closed form, based on the observations of $S_{\tau_1}, S_{\tau_2}, \dots$ for all $\tau_k \leq t$. It turns out that the filter is given by a recursive system of deterministic Kolmogorov-type equations, which should make the numerical implementation relatively easy.

Preliminary data analysis will be discussed.

The talk is based on the papers:

1. J. Cvitanic, R. Liptser, B. Rozovskii. A filtering approach to tracking volatility from prices observed at random times. Annals of Applied Prob, 16 (2006), no. 3, 1633—1652

2. J. Cvitanic, B. Rozovskii, Il. Zalyapin. Numerical estimation of volatility values from discretely observed diffusion data. J. Computational Finance (to appear)