Greed, Leverage, and Potential Losses: A Prospect Theory Perspective

Xunyu Zhou

Based on joint work with Hanqing Jin

28th March 2009/Oxford–Princeton Workshop

Prologue: Financial Crisis Blame Game

Who's to blame for this financial crisis: The top 10 list

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5. Americans
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3. Economists
2. Quants
1. Financial Engineers and Financial Mathematicians (i.e. you and me)

Ultimate Culprits

- "**Greed**, Stupidity, Delusion – and Some More Greed" (John Steele Gordon; *NY Times*, 23 March 2009)
- "Fear, **Greed**, and the Financial Crisis – an American Expat's View From Abroad" (*US News and World Report*, 16 October 2008)
- "Rowan Williams says 'human **greed**' to blame for financial crisis" (*Times Online*, 15 October 2008)
- German finance minister Peer Steinbrück denounces US **greed** (October 2008)
- "...hardwired human behavior coupled with free enterprise and modern capitalism" (Andrew Lo)

Goal of This Work

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- Suggest alternative models where greed is contained (if indirectly)

The Market

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- Arbitrage-free and complete market

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Merton (1971); abundant research thereafter

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  - **Attitude towards risk**: Investors are always risk averse (concave utility)
  - **Beliefs about future**: Investors are able to objectively evaluate probabilities of future returns

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  - People act independently on the basis of full and relevant information

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- Chicago school (Milton Friedman 1912-2006): regulation and other government intervention always inefficient compared to a free market
- *Reaganomics*: "Only by reducing the growth of government, can we increase the growth of the economy"

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  - *Beliefs about future*: Investors *exaggerate* small probabilities

An Experiment on Comparison

- Experiment (Alan Greenspan): compare the following two job offers:
  - A: Earn £120,000/year while all your colleagues earn at *least* £240,000/year
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  - people are born to compare

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The answer is behavioural (yes people are irrational and they compare)!

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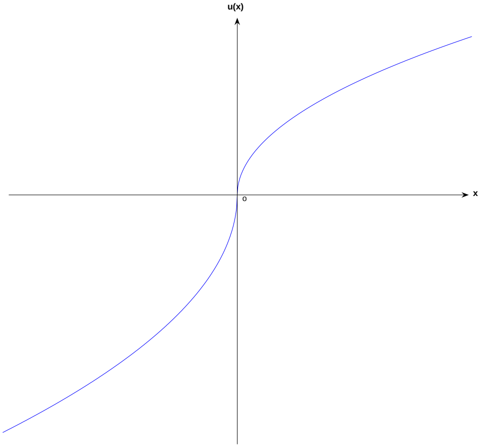
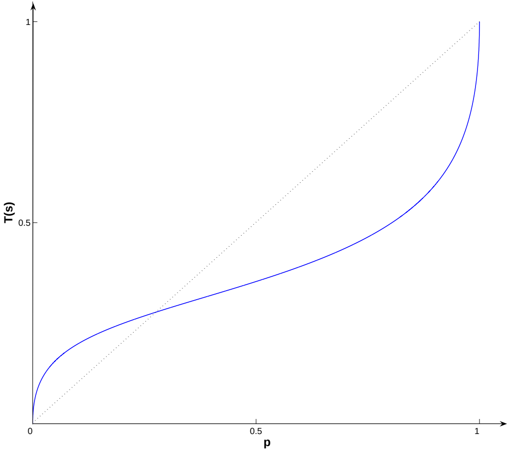
- Probability distortions

S-shaped Function



Probability Distortion Function



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- Utility function

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where $\alpha = \beta = 0.88, \quad k = 2.25$

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- Probability distortion functions

$$T_+(p) = \frac{p^{\gamma}}{(p^{\gamma}+(1-p)^{\gamma})^{1/\gamma}}$$

$$T_-(p) = \frac{p^{\delta}}{(p^{\delta}+(1-p)^{\delta})^{1/\delta}}$$

where $\gamma = 0.61, \quad \delta = 0.69$

Behavioural Portfolio Choice *à la* Prospect Theory

$$\begin{array}{ll} \underset{X}{\text{Max}} & \int_0^\infty T_+\left(P\left(u_+\left((X-B)_+\right)>x\right)\right)dx \\ & \quad -\int_0^\infty T_-\left(P\left(u_-\left((X-B)_-\right)>x\right)\right)dx \\ \text{Subject to} & E[\rho X]=x_0 \end{array}$$

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Berkelaar, Kouwenberg and Post (2004), Jin and Zhou (2008)

Approaches

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  - Nonconcave in X: convex duality fails
  - Nonlinear expectation with Choquet integration: time-consistency or HJB fails

Jin and Zhou's Solution

Assumption. $u_-(\cdot)$ strictly concave at 0; $F^{-1}(z)/T'_+(z)$ non-decreasing in $z \in (0,1]$; $\liminf_{x\to+\infty}\left(\frac{-xu''_+(x)}{u'_+(x)}\right) > 0$;
$E\left[u_+\left((u'_+)^{-1}(\frac{\rho}{T'_+(F(\rho))})\right)T'_+(F(\rho))\right] < +\infty$

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Consider a mathematical programme in (c, x_+):

$$\text{Maximise} \quad E\left[u_+\left((u'_+)^{-1}\left(\frac{\lambda(c,x_+)\rho}{T'_+(F(\rho))}\right)\right)T'_+(F(\rho))\mathbf{1}_{\rho\le c}\right] \\ -u_-(\frac{x_+-(x_0-E[\rho B])}{E[\rho\mathbf{1}_{\rho>c}]})T_-(1-F(c))$$

$$\text{subject to} \quad \begin{cases} \underline{\rho} \le c \le \bar{\rho}, \quad x_+ \ge (x_0 - E[\rho B])^+, \\ x_+ = 0 \text{ when } c = \underline{\rho}, \quad x_+ = x_0 - E[\rho B] \text{ when } c = \bar{\rho}, \end{cases}$$

where $\lambda(c, x_+)$ satisfies $E\left[(u'_+)^{-1}(\frac{\lambda(c,x_+)\rho}{T'_+(F(\rho))})\rho\mathbf{1}_{\rho\le c}\right] = x_+$

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Optimal solution (Jin and Zhou 2008)

$$X^* = \left[(u'_+)^{-1}\left(\frac{\lambda\rho}{T'_+(F(\rho))}\right)+B\right]\mathbf{1}_{\rho\le c^*} - \left[\frac{x^*_+-(x_0-E[\rho B])}{E[\rho\mathbf{1}_{\rho>c^*}]}-B\right]\mathbf{1}_{\rho>c^*}$$

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- Magnitude of potential losses *dependent* of B

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- greed becomes relevant and significant only when $x_0 < E[\rho B]$
- The higher the reference point the more likely the agent is to be a risk-taker
- a natural definition is the ratio between what the agent is desperate to achieve and what she has to start with

$$G = \frac{E[\rho B]}{x_0}$$

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- a natural definition is the ratio between what the agent is desperate to achieve and what she has to start with

$$G = \frac{E[\rho B]}{x_0}$$

- Definition does not work for expected utility model

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Two-Piece Power Utility

Hereafter we consider $\log \rho \sim N(\mu, \sigma^2)$ with $\sigma > 0$ and

$$u_+(x) = x^\alpha, u_-(x) = k_- x^\beta,\ x \geq 0$$

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Denote

$$\varphi(c) := E\left[\left(\frac{T'_+(F(\rho))}{\rho}\right)^{1/(1-\alpha)} \rho \mathbf{1}_{\rho \leq c}\right] \geq 0, \ \ 0 \leq c \leq +\infty.$$

$$k(c) := \frac{k_- T_-(1 - F(c))}{\varphi(c)^{1-\alpha}(E[\rho \mathbf{1}_{\rho > c}])^\beta} > 0, \ \ c > 0.$$

Case $\alpha = \beta$: Optimal Terminal Wealth

Theorem. (Jin and Zhou 2008) If $\alpha = \beta$ and $x_0 < E[\rho B]$, then behavioural portfolio selection problem has a finite optimal portfolio if and only if $\inf_{c>0} k(c) > 1$ and

$$\operatorname{argmin}_{c\geq 0}\left[\left(\frac{k_- T_-(1-F(c))}{(E[\rho \mathbf{1}_{\rho>c}])^{\alpha}}\right)^{1/(1-\alpha)} - \varphi(c)\mathbf{1}_{c>0}\right] \neq \emptyset. \quad (1)$$

Moreover, if $c^* > 0$ is one of the minimizers in (1), then optimal terminal wealth is

$$X^* = \frac{x_+^*}{\varphi(c^*)}\left(\frac{T_+'(F(\rho))}{\rho}\right)^{1/(1-\alpha)} \mathbf{1}_{\rho\leq c^*} - \frac{x_+^* - (x_0 - E[\rho B])}{E[\rho \mathbf{1}_{\rho>c^*}]}\mathbf{1}_{\rho>c^*} + B,$$

where $x_+^* := \frac{-(x_0 - E[\rho B])}{k(c^*)^{1/(1-\alpha)} - 1}$; and if $c^* = 0$ is the unique minimizer in (1), then optimal terminal wealth is $X^* = \frac{x_0 - E[\rho B]}{E\rho} + B$.

Case $\alpha = \beta$: Leverage and Greed

Assume that $x_0 < E(\rho B)$ and that $c^* > 0$ is one of the minimizers in (1)

- $X_l^* = \left(\frac{x_+^* - (x_0 - E[\rho B])}{E[\rho \mathbf{1}_{\rho > c^*}]} - B \right) \mathbf{1}_{\rho > c^*}$

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$$\frac{x_+^* - (x_0 - E[\rho B])}{E[\rho \mathbf{1}_{\rho > c^*}]} - B = \left(\frac{aE[\rho B]}{E[\rho \mathbf{1}_{\rho > c^*}]} - B\right) - \frac{ax_0}{E[\rho \mathbf{1}_{\rho > c^*}]}$$

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- Then the leverage

$$\begin{aligned} L &= \frac{E(\rho X_l^*)}{x_0} = \frac{1}{x_0} E\left[\rho \left(\frac{x_+^* - (x_0 - E[\rho B])}{E[\rho \mathbf{1}_{\rho > c^*}]} - B \right) \mathbf{1}_{\rho > c^*} \right] \\ &\geq (a-1)\frac{E(\rho B)}{x_0} - a \\ &= (a-1)G - a \to +\infty \text{ as } G \to +\infty. \end{aligned}$$

Case $\alpha = \beta$: Potential Losses and Greed

The potential loss

$$
\begin{aligned}
l &= E\left(\frac{\rho X_l^*}{x_0}\middle| X^* < B\right) = E\left(\frac{\rho X_l^*}{x_0}\middle| \rho > c^*\right) \\
&= \frac{E(\frac{\rho X_l^*}{x_0})}{P(\rho > c^*)} \\
&\geq \frac{(a-1)G - a}{P(\rho > c^*)} \to +\infty \text{ as } G \to +\infty.
\end{aligned}
$$

Case $\alpha = \beta$: Results

Theorem. (Jin and Zhou 2009) Assume that $x_0 < E(\rho B)$ and that $c^* > 0$ is one of the minimizers in (1). Then we have the following conclusions:

(i) $L \to +\infty$ as $G \to +\infty$.

(ii) $P(X^* < B) \equiv P(\rho > c^*)$ is *independent* of G.

(iii) $l \to +\infty$ as $G \to +\infty$.

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- Probability of loss occurrence now depends on B or G

Case $\alpha < \beta$: A Critical Point

Denote

$$h(c) := \frac{k_- T_-(1-F(c))}{(E[\rho \mathbf{1}_{\rho > c}])^\beta}, \quad c > 0$$

$$c_1 := \sup\{c' \in [0, +\infty) : h(c') = \inf_{c \in [0,+\infty)} h(c)\}$$

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- Portfolio problem admits no optimal solution if $c_1 = +\infty$

Case $\alpha < \beta$: Results

Theorem. (Jin and Zhou 2009) Assume that $x_0 < E(\rho B)$, $\liminf_{c \to +\infty} h(c) > 0$, and $0 < c_1 < +\infty$. Then portfolio problem admits optimal solution with a sufficiently large agent greed G. Furthermore, if $(c(G), x_+(G))$ is an optimal solution for the mathematical programme, then optimal terminal wealth is

$$X^* = \frac{x_+(G)}{\varphi(c(G))} \left(\frac{T'_+(F(\rho))}{\rho} \right)^{1/(1-\alpha)} \mathbf{1}_{\rho \le c(G)} - \frac{x_+(G) - (x_0 - E[\rho B])}{E[\rho \mathbf{1}_{\rho > c(G)}]} \mathbf{1}_{\rho > c(G)} + B.$$

Moreover,

$$\lim_{G \to +\infty} c(G) = c_1, \quad \lim_{G \to +\infty} x_+(G) = +\infty, \quad \lim_{G \to +\infty} \frac{x_+(G)}{G} = 0.$$

Finally, $L \to +\infty$ as $G \to +\infty$ and $l \to +\infty$ as $G \to +\infty$.

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- The new model (Jin, Zhang, Zhou 2009)

$$\begin{array}{ll} \text{Maximize} & V(X-B) \\ \text{subject to} & \begin{cases} E[\rho X] = x_0 \\ X \geq B - L \\ X \text{ is an } \mathcal{F}_T - \text{random variable} \end{cases} \end{array}$$

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- Behavioural finance a promising area in helping with re-building sound post-crisis financial infrastructure