

# Cap-and-Trade Schemes for the Emissions Markets: Design, Calibration and Option Pricing

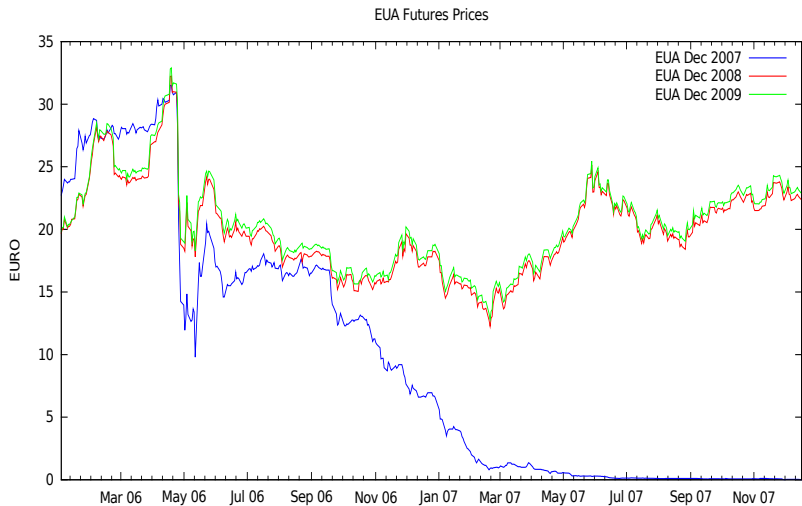
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Princeton March 27, 2009

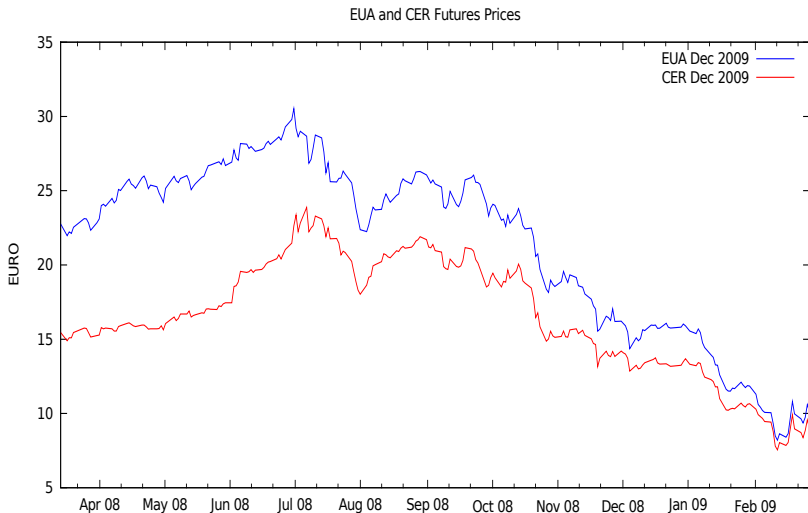
- **Cap & Trade Schemes for CO<sub>2</sub> Emissions**
  - Kyoto Protocol
  - Mandatory Carbon Markets (**EU ETS, RGGI since 01/01/09**)
  - Lessons learned from the EU Experience
- **Mathematical (Equilibrium) Models**
  - Price Formation for Goods and Emission Allowances
  - New Designs and Alternative Schemes
  - Calibration & Option Pricing
- **Computer Implementations**
  - Several case studies (Texas, Japan)
  - Practical Tools for Regulators and Policy Makers

- **No (Significant) Emissions Reduction**
  - DID Emissions go down?
  - Yes, but as part of an existing trend
- **Significant Increase in Prices**
  - Cost of Pollution passed along to the "end-consumer"
  - Small proportion (40%) of polluters involved in EU ETS
- **Windfall Profits**
  - Cannot be avoided
  - Proposed Remedies
    - Stop Giving Allowance Certificates Away for Free !
    - Auctioning

# Falling Carbon Prices: What Happened?



# CDM: Can we Explain CER Prices?



# Description of the Economy

- **Finite set**  $\mathcal{I}$  of **risk neutral firms**
- **Producing a finite set**  $\mathcal{K}$  of **goods**
- Firm  $i \in \mathcal{I}$  can use **technology**  $j \in \mathcal{J}^{i,k}$  to produce good  $k \in \mathcal{K}$
- **Discrete time**  $\{0, 1, \dots, T\}$
- **No Discounting** Work with  $T$ -Forward Prices
- **Inelastic Demand**

$$\{D^k(t); t = 0, 1, \dots, T - 1, k \in \mathcal{K}\}.$$

● .....

# Regulator Input (EU ETS)

At inception of program (i.e. time  $t = 0$ )

- **INITIAL DISTRIBUTION** of **allowance certificates**

$$\theta_0^i \quad \text{to firm } i \in \mathcal{I}$$

- Set **PENALTY**  $\pi$  for emission unit **NOT** offset by allowance certificate at end of **compliance period**

Extensions (not discussed in this talk)

- **Risk aversion** and agent preferences (existence theory easy)
- **Elastic** demand (e.g. smart meters for electricity)
- **Multi-period models** with lending, borrowing and withdrawal (more realistic)
- .....

# Goal of Equilibrium Analysis

Find **two stochastic processes**

- **Price of one allowance**

$$A = \{A_t\}_{t \geq 0}$$

- **Prices of goods**

$$S = \{S_t^k\}_{k \in K, t \geq 0}$$

satisfying the usual conditions for the existence of a

***competitive equilibrium***

(to be spelled out below).

# Individual Firm Problem

During each time period  $[t, t + 1)$

- Firm  $i \in \mathcal{I}$  **produces**  $\xi_t^{i,j,k}$  of good  $k \in \mathcal{K}$  with technology  $j \in \mathcal{J}^{i,k}$
- Firm  $i \in \mathcal{I}$  **holds** a position  $\theta_t^i$  in emission credits

$$\begin{aligned} L^{A,S,i}(\theta^i, \xi^i) := & \sum_{k \in \mathcal{K}} \sum_{j \in \mathcal{J}^{i,k}} \sum_{t=0}^{T-1} (S_t^k - C_t^{i,j,k}) \xi_t^{i,j,k} \\ & + \theta_0^i A_0 + \sum_{t=0}^{T-1} \theta_{t+1}^i (A_{t+1} - A_t) - \theta_{T+1}^i A_T \\ & - \pi(\Gamma^i + \Pi^i(\xi^i) - \theta_{T+1}^i)^+ \end{aligned}$$

where

$$\Gamma^i \text{ random, } \quad \Pi^i(\xi^i) := \sum_{k \in \mathcal{K}} \sum_{j \in \mathcal{J}^{i,k}} \sum_{t=0}^{T-1} e^{i,j,k} \xi_t^{i,j,k}$$

**Problem for (risk neutral) firm  $i \in \mathcal{I}$**

$$\max_{(\theta^i, \xi^i)} \mathbb{E}\{L^{A,S,i}(\theta^i, \xi^i)\}$$

# In the Absence of Cap-and-Trade Scheme (i.e. $\pi = 0$ )

If  $(A^*, S^*)$  is an equilibrium, the optimization problem of firm  $i$  is

$$\sup_{(\theta^i, \xi^i)} \mathbb{E} \left[ \sum_{k \in \mathcal{K}} \sum_{j \in \mathcal{J}^{i,k}} \sum_{t=0}^{T-1} (S_t^k - C_t^{i,j,k}) \xi_t^{i,j,k} + \theta_0^i A_0 + \sum_{t=0}^{T-1} \theta_{t+1}^i (A_{t+1} - A_t) - \theta_{T+1}^i A_T \right]$$

We have  $A_t^* = \mathbb{E}_t[A_{t+1}^*]$  for all  $t$  and  $A_T^* = 0$  (hence  $A_t^* \equiv 0$ !)

Classical competitive equilibrium problem where each agent maximizes

$$\sup_{\xi^i \in \mathcal{U}^i} \mathbb{E} \left[ \sum_{k \in \mathcal{K}} \sum_{j \in \mathcal{J}^{i,k}} \sum_{t=0}^{T-1} (S_t^k - C_t^{i,j,k}) \xi_t^{i,j,k} \right], \quad (1)$$

and the equilibrium prices  $S^*$  are set so that supply meets demand. For each time  $t$

$$((\xi_t^{*,i,j,k})_{j,k})_i = \arg \max_{((\xi_t^{i,j,k})_{\mathcal{J}^{i,k}})_{i \in \mathcal{I}}} \sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}^{i,k}} -C_t^{i,j,k} \xi_t^{i,j,k}$$

$$\sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}^{i,k}} \xi_t^{i,j,k} = D_t^k$$

$$\xi_t^{i,j,k} \leq \kappa^{i,j,k} \quad \text{for } i \in \mathcal{I}, j \in \mathcal{J}^{i,k}$$

$$\xi_t^{i,j,k} \geq 0 \quad \text{for } i \in \mathcal{I}, j \in \mathcal{J}^{i,k}$$

The corresponding prices of the goods are

$$S_t^{*k} = \max_{i \in \mathcal{I}, j \in \mathcal{J}^{i,k}} C_t^{i,j,k} \mathbf{1}_{\{\xi_t^{*i,j,k} > 0\}},$$

## Classical **MERIT ORDER**

- At each time  $t$  and for each good  $k$
- Production technologies ranked by increasing production costs  $C_t^{i,j,k}$
- Demand  $D_t^k$  met by producing from the cheapest technology first
- Equilibrium spot price is the marginal cost of production of the most expansive production technology used to meet demand

### **Business As Usual**

(typical scenario in Deregulated **electricity markets**)

# Equilibrium Definition for Emissions Market

The processes  $A^* = \{A_t^*\}_{t=0,1,\dots,T}$  and  $S^* = \{S_t^*\}_{t=0,1,\dots,T}$  form an equilibrium if for each agent  $i \in \mathcal{I}$  there exist strategies  $\theta^{*i} = \{\theta_t^{*i}\}_{t=0,1,\dots,T}$  (**trading**) and  $\xi^{*i} = \{\xi_t^{*i}\}_{t=0,1,\dots,T}$  (**production**)

- **(i) All financial positions are in constant net supply**

$$\sum_{i \in I} \theta_t^{*i} = \sum_{i \in I} \theta_0^i, \quad \forall t = 0, \dots, T + 1$$

- **(ii) Supply meets Demand**

$$\sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}^{i,k}} \xi_t^{*i,j,k} = D_t^k, \quad \forall k \in \mathcal{K}, t = 0, \dots, T - 1$$

- **(iii) Each agent  $i \in I$  is satisfied by its own strategy**

$$\mathbb{E}[L^{A^*, S^*, i}(\theta^{*i}, \xi^{*i})] \geq \mathbb{E}[L^{A^*, S^*, i}(\theta^i, \xi^i)] \quad \text{for all } (\theta^i, \xi^i)$$

# Necessary Conditions

## Assume

- $(A^*, S^*)$  is an equilibrium
- $(\theta^{*i}, \xi^{*i})$  optimal strategy of agent  $i \in I$

## then

- The allowance price  $A^*$  is a **bounded martingale** in  $[0, \pi]$
- Its terminal value is given by

$$A_T^* = \pi \mathbf{1}_{\{\Gamma^i + \Pi(\xi^{*i}) - \theta_{T+1}^{*i} \geq 0\}} = \pi \mathbf{1}_{\{\sum_{i \in I} (\Gamma^i + \Pi(\xi^{*i}) - \theta_{T+1}^{*i}) \geq 0\}}$$

- The **spot prices**  $S^{*k}$  of the goods and the **optimal production strategies**  $\xi^{*i}$  are given by the **merit order** for the equilibrium with **adjusted costs**

$$\tilde{C}_t^{i,j,k} = C_t^{i,j,k} + e^{i,j,k} A_t^*$$

# Social Cost Minimization Problem

- Overall production costs

$$C(\xi) := \sum_{t=0}^{T-1} \sum_{(i,j,k)} \xi_t^{i,j,k} C_t^{i,j,k}.$$

- Overall cumulative emissions

$$\Gamma := \sum_{i \in I} \Gamma^i \quad \Pi(\xi) := \sum_{t=0}^{T-1} \sum_{(i,j,k)} e^{i,j,k} \xi_t^{i,j,k},$$

- Total allowances

$$\theta_0 := \sum_{i \in I} \theta_0^i$$

The **total social costs from production and penalty payments**

$$G(\xi) := C(\xi) + \pi(\Gamma + \Pi(\xi) - \theta_0)^+$$

We introduce the global optimization problem

$$\xi^* = \arg \inf_{\xi \text{ meets demands}} \mathbb{E}[G(\xi)],$$

- **First Theoretical Result**

- There exists a set  $\xi^* = (\xi^{*i})_{i \in I}$  realizing the minimum social cost

- **Second Theoretical Result**

- (i) If  $\bar{\xi}$  minimizes the social cost, then the processes  $(\bar{A}, \bar{S})$  defined by

$$\bar{A}_t = \pi \mathbb{P}_t \{ \Gamma + \Pi(\bar{\xi}) - \theta_0 \geq 0 \}, \quad t = 0, \dots, T$$

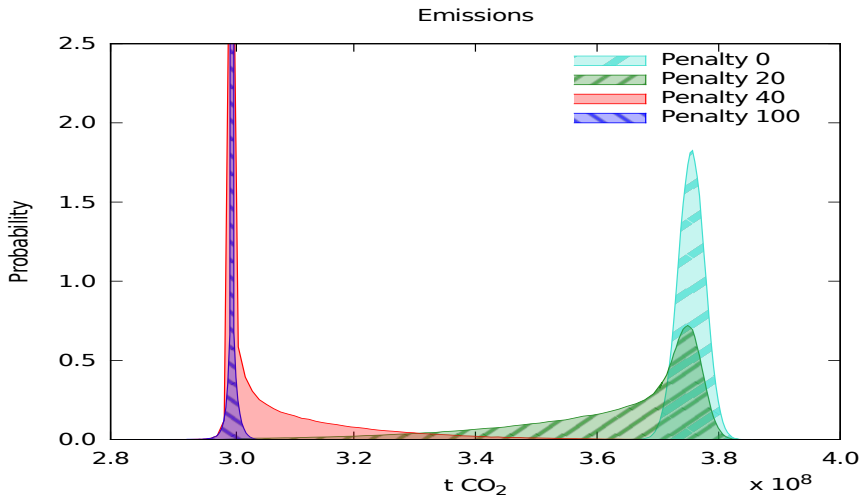
and

$$\bar{S}_t^k = \max_{i \in I, j \in J^{i,k}} (C_t^{i,j,k} + e_t^{i,j,k} \bar{A}_t) 1_{\{\bar{\xi}_t^{i,j,k} > 0\}}, \quad t = 0, \dots, T-1 \quad k \in K,$$

form a **market equilibrium** with associated production strategy  $\bar{\xi}$

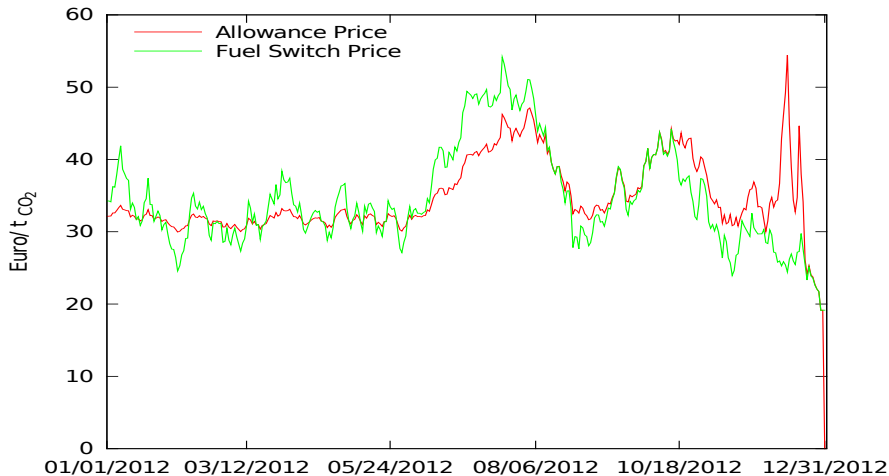
- (ii) If  $(A^*, S^*)$  is an equilibrium with corresponding strategies  $(\theta^*, \xi^*)$ , then  $\xi^*$  solves the **social cost minimization problem**
- (iii) The equilibrium allowance price is **unique**.

# Effect of the Penalty on Emissions



# Price Equilibrium Sample Path

Simulated Price Paths



# Costs in a Cap-and-Trade

- **Consumer Burden**

$$SC = \sum_t \sum_k (S_t^{k,*} - S_t^{k,BAU*}) D_t^k.$$

- **Reduction Costs** (producers' burden)

$$\sum_t \sum_{i,j,k} (\xi_t^{i,j,k*} - \xi_t^{BAU,i,j,k*}) C_t^{i,j,k}$$

- **Excess Profit**

$$\sum_t \sum_k (S_t^{k,*} - S_t^{k,BAU*}) D_t^k - \sum_t \sum_{i,j,k} (\xi_t^{i,j,k*} - \xi_t^{BAU,i,j,k*}) C_t^{i,j,k} - \pi \left( \sum_t \sum_{ijk} \xi_t^{ijk} e_t^{ijk} - \theta_0 \right)^+$$

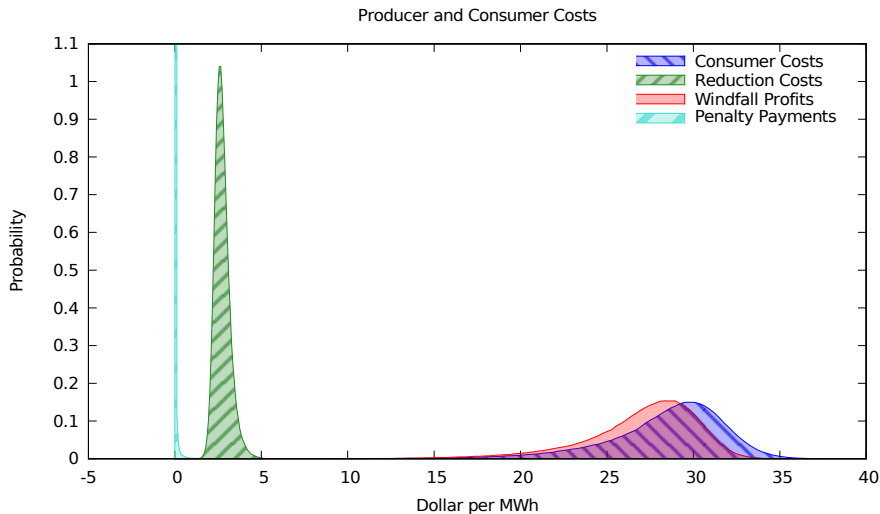
- **Windfall Profits**

$$WP = \sum_{t=0}^{T-1} \sum_{k \in K} (S_t^{*k} - \hat{S}_t^k) D_t^k$$

where

$$\hat{S}_t^k := \max_{i \in I, j \in J^{i,k}} C_t^{i,j,k} \mathbf{1}_{\{\xi_t^{*i,j,k} > 0\}}.$$

# Costs in a Cap-and-Trade Scheme



Histograms of consumer costs, social costs, windfall profits and penalty payments of a standard cap-and-trade scheme calibrated to reach the emissions target with 95% probability and BAU.

# One of many Possible Generalizations

## Introduction of **Taxes / Subsidies**

$$\begin{aligned} \ddot{L}^{A,S,i}(\theta^i, \xi^i) = & - \sum_{t=0}^{T-1} G_t^i + \sum_{k \in K} \sum_{j \in J^{i,k}} \sum_{t=0}^{T-1} (S_t^k - C_t^{i,j,k} - H_t^k) \xi_t^{i,j,k} \\ & + \sum_{t=0}^{T-1} \theta_t^i (A_{t+1} - A_t) - \theta_T^i A_T \\ & - \pi(\Gamma^i + \Pi^i(\xi^i) - \theta_T^i)^+. \end{aligned}$$

In this case

- In equilibrium, **production** and **trading** strategies remain the same  $(\theta^\dagger, \xi^\dagger) = (\theta^*, \xi^*)$
- **Abatement costs** and **Emissions reductions** are also the same
- New equilibrium prices  $(A^\dagger, S^\dagger)$  given by

$$A_t^\dagger = A_t^* \quad \text{for all } t = 0, \dots, T \quad (2)$$

$$S_t^{\dagger k} = S_t^{*k} + H_t^k \quad \text{for all } k \in K, t = 0, \dots, T-1 \quad (3)$$

- Cost of the tax passed along to the end consumer

- **Currently Regulator Specifies**

- Penalty  $\pi$
- Overall Certificate Allocation  $\theta_0 (= \sum_{i \in I} \theta_0^i)$

- **Alternative Scheme (Still) Controlled by Regulator**

(i) **Sets penalty level**  $\pi$

(ii) **Allocates allowances**

- $\theta_0^i$  at inception of program  $t = 0$
- then **proportionally to production**

$y \xi_t^{i,j,k}$  to agent  $i$  for producing  $\xi_t^{i,j,k}$  of good  $k$  with technology  $j$

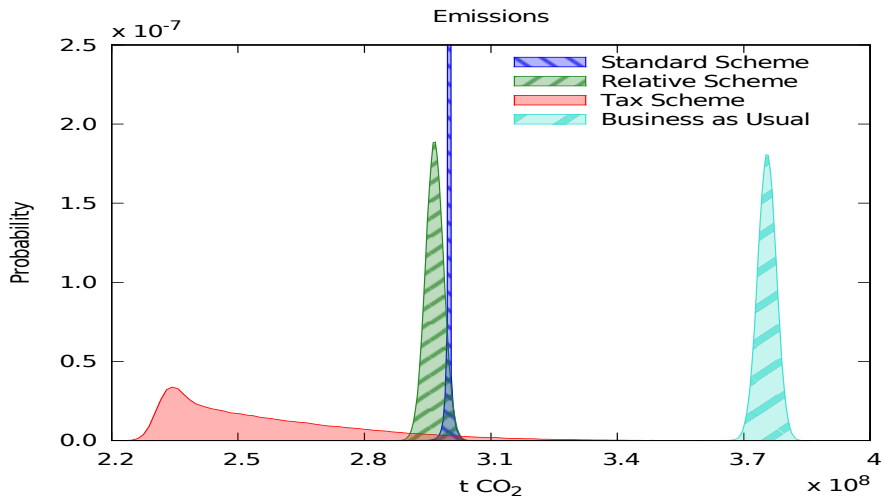
(iii) **Calibrates**  $y$ , e.g. in **expectation**.

$$y = \frac{\theta_0 - \theta_0'}{\sum_{t=0}^{T-1} \sum_{k \in K} \mathbb{E}\{D_t^k\}}$$

So total number of credit allowance is the same in expectation, i.e.

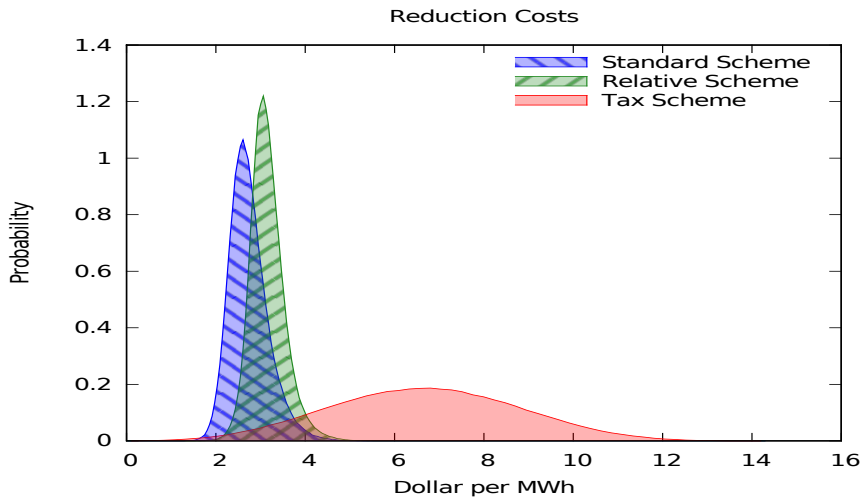
$$\theta_0 = \mathbb{E}\{\theta_0' + y \sum_{t=0}^{T-1} \sum_{k \in K} D_t^k\}$$

# Yearly Emissions Equilibrium Distributions



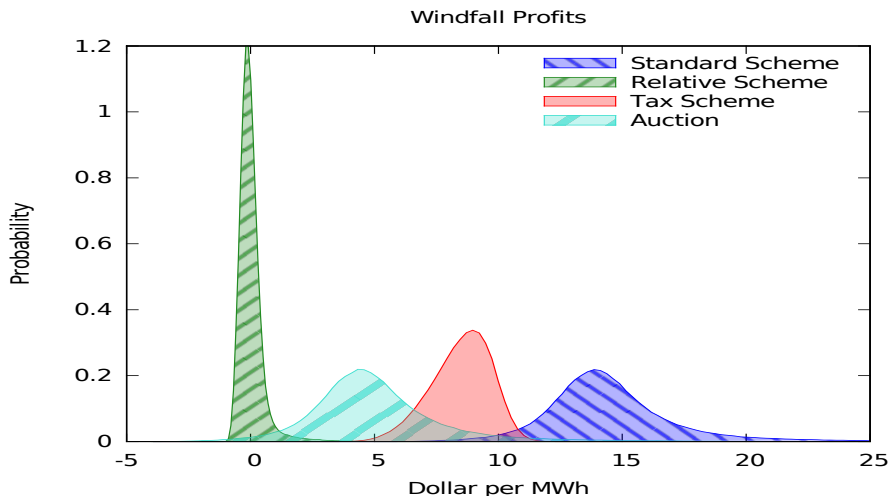
Yearly emissions from electricity production for the Standard Scheme, the Relative Scheme, a Tax Scheme and BAU.

# Abatement Costs



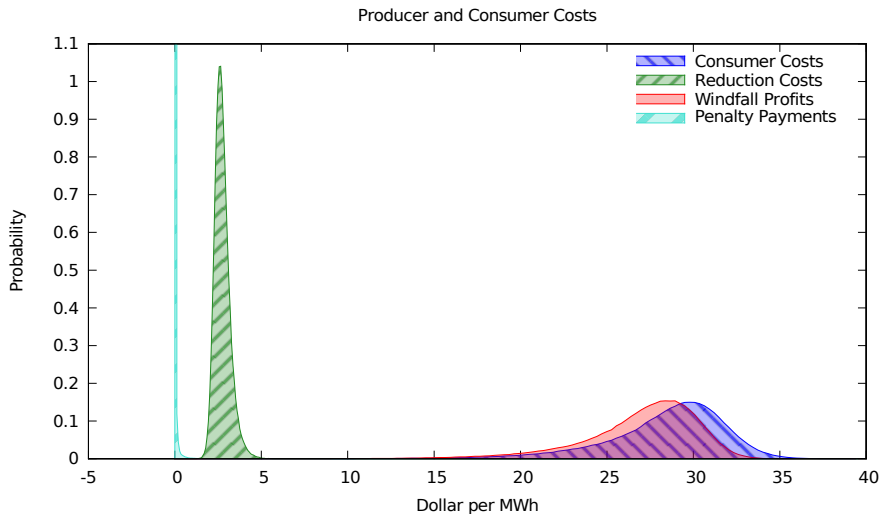
Yearly abatement costs for the Standard Scheme, the Relative Scheme and a Tax Scheme.

# Windfall Profits



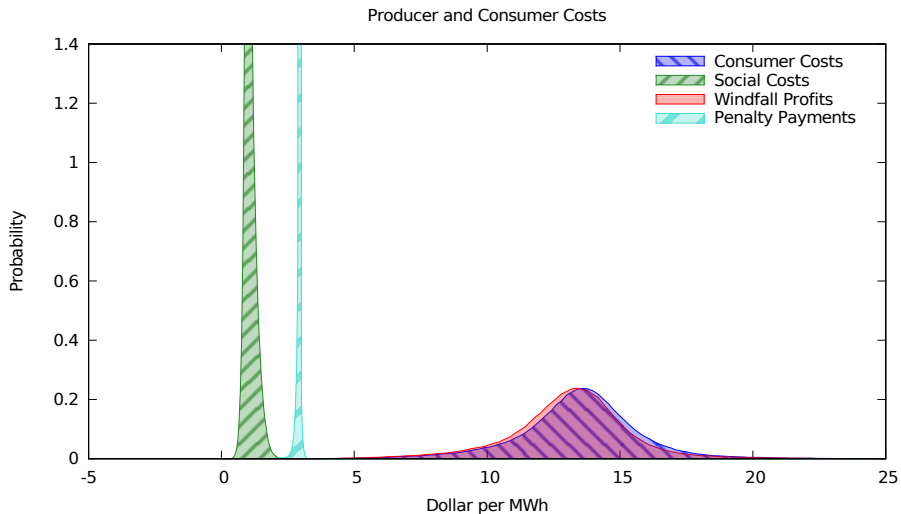
Histograms of the yearly distribution of windfall profits for the Standard Scheme, a Relative Scheme, a Standard Scheme with 100% Auction and a Tax Scheme

# Japan Case Study: Windfall Profits



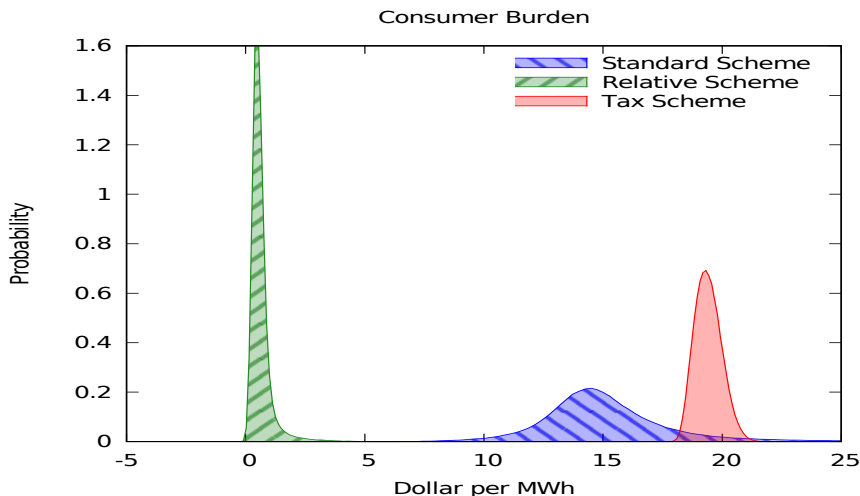
Histograms of the difference of consumer cost, social cost, windfall profits and penalty payments between BAU and a standard trading scheme scenario with a cap of 300Mt CO<sub>2</sub>. Notice that taking into account fuel switching even

# Japan Case Study: More Windfall Profits



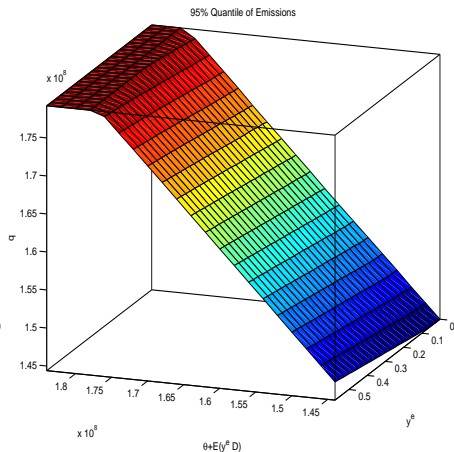
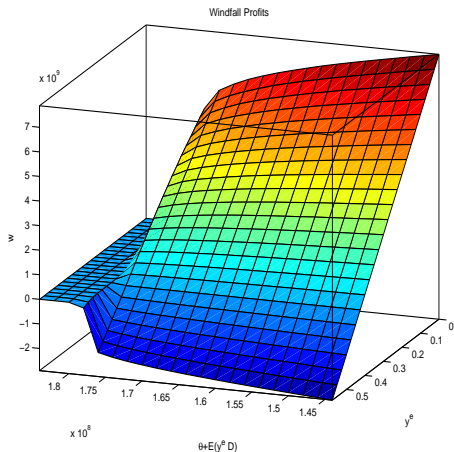
Histograms of the consumer cost, social cost, windfall profits and penalty payments under a standard trading scheme scenario with a cap of  $330\text{MtCO}_2$ .

# Japan Case Study: Consumer Costs



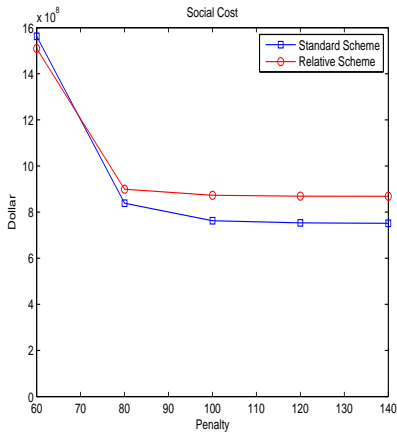
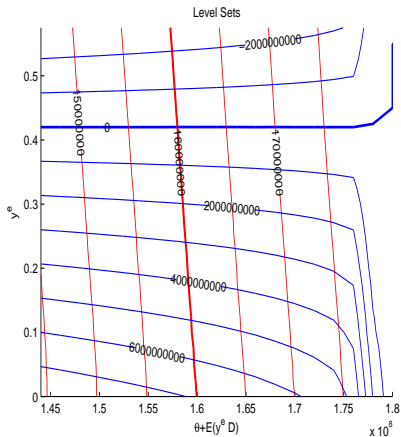
Histogram of the yearly distribution of consumer costs for the Standard Scheme, a Relative Scheme and a Tax Scheme. Notice that the Standard Scheme with Auction possesses the same consumer costs as the Standard

# Numerical Results: Windfall Profits



Windfall profits (left) and 95% percentile of total emissions (right) as functions of the relative allocation parameter and the expected allocation

# More Numerical Results: Windfall Profits



(left) Level sets of previous plots. (right) Production costs for electricity for one year as function of the penalty level for both the absolute and relative schemes.

# Equilibrium Models: (Temporary) Conclusions

- Market Mechanisms **CANNOT** solve all the pollution problems
- **Cap-and-Trade Schemes CAN Work!**
  - Given the right emission target
  - Using the appropriate tool to allocate emissions credits
  - Significant Windfall Profits for Standard Schemes
- **Taxes**
  - Politically unpopular
  - Cannot reach emissions targets
- **Auctioning**
  - Fairness is Smoke Screen: Re-distribution of the cost
- **Relative Schemes**
  - **Pros**
    - Can Reach Emissions Target (statistics)
    - Possible Control of Windfall Profits
    - Minimize Social Costs
  - **Cons**
    - Number of Allowances **NOT** exactly known in advance
- **Mixed Scheme (Relative Scheme + Auction**
  - Same Pros as Relative Scheme
  - Number of Allowances **FIXED** in advance

# Reduced Form Models & Option Pricing

- Emissions Cap-and-Trade Markets **SOON** to exist in the US
- **Option** Market **SOON** to develop
  - Underlying  $\{A_t\}_t$  non-negative martingale with **binary terminal value**
  - Can think of  $A_t$  as of a binary option
  - Underlying of binary option should be *Emissions*
- Need for **Formulae** (closed or computable)
  - for Prices
  - for Hedges
- **Reduced Form Models**

# Reduced Form Model for Emissions Abatement

- $\{X_t\}_t$  actual emissions at time  $t$   
 $dX_t = \sigma(t, X_t)dW_t - \xi_t dt$ 
  - $\xi_t$  abatement (in ton of  $CO_2$ ) at time  $t$
  - $X_t = E_t - \int_0^t \xi_s ds$   
cumulative emissions in BAU minus abatement up to time  $t$
- $\pi(X_T - K)^+$  penalty
  - $T$  maturity (end of compliance period)
  - $K$  regulator emissions' target
  - $\pi$  penalty (40 EURO) per ton of  $CO_2$  not offset by an allowance certificate
- **Social Cost**  $\mathbb{E}\{\int_0^T C(\xi_s) ds + \pi(X_T - K)^+\}$ 
  - $C(\xi)$  cost of abatement of  $\xi$  ton of  $CO_2$

## Informed Planner Problem

$$\inf_{\xi=\{\xi_t\}_{0\leq t\leq T}} \mathbb{E}\left\{\int_0^T C(\xi_s)ds + \pi(X_T - K)^+\right\}$$

## Value Function

$$V(t, x) = \inf_{\{\xi_s\}_{t\leq s\leq T}} \mathbb{E}\left\{\int_t^T C(\xi_s)ds + \pi(X_T - K)^+ | X_t = x\right\}$$

## HJB equation (e.g. $C(\xi) = \xi^2$ )

$$V_t + \frac{1}{2}\sigma(t, x)^2 V_{xx} - \frac{1}{2}V_x^2$$

## Emission Allowance Price

$$A_t = V_x(t, X_t)$$

## Emission Allowance Volatility

$$\sigma_A(t) = \sigma(t, X_t) V_{xx}(t, X_t)$$

## Calibration ( $\sigma(t)$ deterministic)

- Multiperiod (**Cetin. et al**)
- Close Form Formulae for Prices
- Close Form Formulae for Hedges

$$A_t = \pi \mathbb{E}\{1_N | \mathcal{F}_t\}, \quad t \in [0, T]$$

Non-compliance  $N \in \mathcal{F}_T$  when a hypothetic positive random variable  $\Gamma_T$  (normalized total emissions) exceeds 1.

$$N = \{\Gamma_T \geq 1\}$$

So

$$A_t = \pi \mathbb{E}1_{\{\Gamma_T \geq 1\}} | \mathcal{F}_t\}, \quad t \in [0, T].$$

Pick  $\Gamma_T$  from a parametric family. Set

$$a_t = \frac{1}{\pi} A_t \quad \text{and choose} \quad \Gamma_T = \Gamma_0 e^{\int_0^T \sigma_s dW_s - \frac{1}{2} \int_0^T \sigma_s^2 ds},$$

for some square-integrable and deterministic function

$$(0, T) \ni t \mapsto \sigma_t.$$

$a_t$  is given by

$$a_t = \Phi \left( \frac{\Phi^{-1}(a_0) \sqrt{\int_0^T \sigma_s^2 ds} + \int_0^t \sigma_s dW_s}{\sqrt{\int_t^T \sigma_s^2 ds}} \right) \quad t \in [0, T]$$

where  $\Phi$  is standard normal c.d.f..

$a_t$  solves the SDE

$$da_t = \Phi'(\Phi^{-1}(a_t)) \sqrt{z_t} dW_t$$

where the positive-valued function  $(0, T) \ni t \mapsto z_t$  is given by

$$z_t = \frac{\sigma_t^2}{\int_t^T \sigma_u^2 du}, \quad t \in (0, T)$$

$$\{z_t(\alpha, \beta) = \beta(T-t)^{-\alpha}\}_{t \in [0, T]}, \quad \beta > 0, \alpha \geq 1. \quad (4)$$

$\beta$  is a multiplicative parameter

$$z_t(\alpha, \beta) = \beta z_t(\alpha, 1), \quad t \in (0, T), \quad \beta > 0, \quad \alpha \geq 1. \quad (5)$$

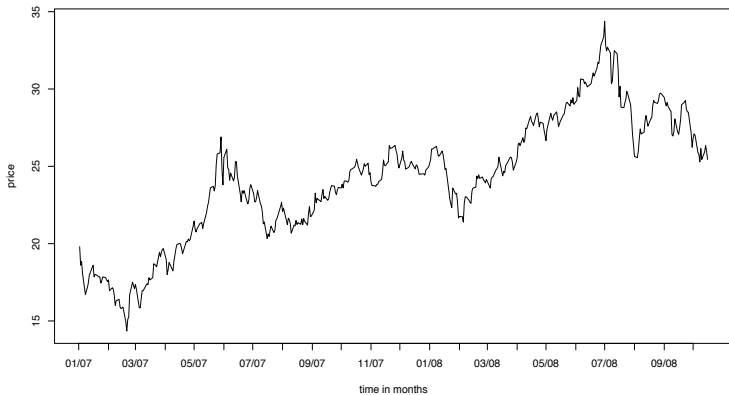
The function  $\{\sigma_t(\alpha, \beta)\}_{t \in (0, T)}$  is given by

$$\sigma_t(\alpha, \beta)^2 = z_t(\alpha, \beta) e^{-\int_0^t z_u(\alpha, \beta) du} \quad (6)$$

$$= \begin{cases} \beta(T-t)^{-\alpha} e^{\beta \frac{T^{-\alpha+1} - (T-t)^{-\alpha+1}}{-\alpha+1}} & \text{for } \beta > 0, \alpha > 1 \\ \beta(T-t)^{\beta-1} T^{-\beta} & \text{for } \beta > 0, \alpha = 1 \end{cases} \quad (7)$$

## Maximum Likelihood

# Sample Data



**Figure:** Future prices on EUA with maturity Dec. 2012

# Call Option Price in One Period Model

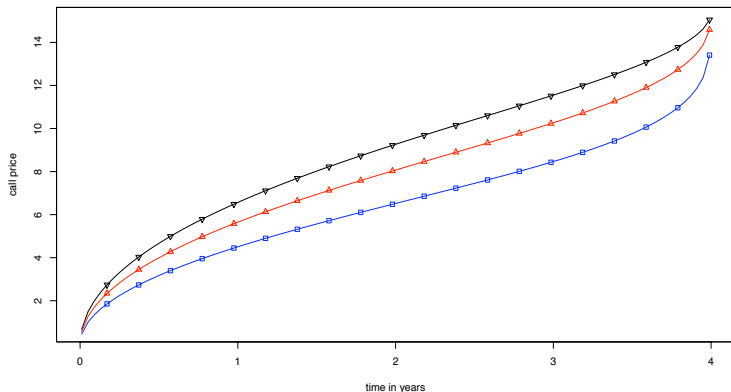
for  $\alpha = 1$ ,  $\beta > 0$ , the price of an European call with strike price  $K \geq 0$  written on a one-period allowance futures price at time  $\tau \in [0, T]$  is given at time  $t \in [0, \tau]$  by

$$\begin{aligned}C_t &= e^{-\int_t^\tau r_s ds} \mathbb{E}\{(A_\tau - K)^+ | \mathcal{F}_t\} \\ &= \int (\pi \Phi(x) - K)^+ N(\mu_{t,\tau}, \nu_{t,\tau})(dx)\end{aligned}$$

where

$$\begin{aligned}\mu_{t,\tau} &= \Phi^{-1}(A_t/\pi) \sqrt{\left(\frac{T-t}{T-\tau}\right)^\beta} \\ \nu_{t,\tau} &= \left(\frac{T-t}{T-\tau}\right)^\beta - 1.\end{aligned}$$

# Price Dependence on $T$ and Sensitivity to $\beta$



**Figure:** Dependence  $\tau \mapsto C_0(\tau)$  of Call prices on maturity  $\tau$ . Graphs  $\square$ ,  $\triangle$ , and  $\nabla$  correspond to  $\beta = 0.5$ ,  $\beta = 0.8$ ,  $\beta = 1.1$ .

## ● Emissions Markets

- R.C., M. Fehr and J. Hinz: Mathematical Equilibrium and Market Design for Emissions Markets Trading Schemes. *SIAM J. Control and Optimization* (2009)
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