5th Oxford-Princeton Workshop on Financial Mathematics & Stochastic Analysis

March 27-28, 2009, Princeton University

Schedule of Talks

All talks will be held in *Sherrerd Hall Room 101.*

FRIDAY 3/27	
8:00 - 9:00am	`Breakfast & Opening Remarks`
9:00 - 9:40	René Carmona
9:40 - 10:20	Sam Howison
10:20 - 10:50	`Coffee Break`
10:50 - 11:30	Dmitry Kramkov
11:30 - 12:10	Ramon van Handel
12:10 - 12:30	Sergey Nadtochiy
12.30 - 12.50	Gechun Liang
12:50 - 2:20pm	`Lunch` *(Sherrerd Hall Atrium)*
2:20 - 3:00	Paul Glasserman
3:00 - 3:40	Jakub Jurek
3:40 - 4:10	`Coffee Break`
4:10 - 4:50	Jan Obloj
4:50 - 5:10	Forrest Zhang
5.10 - 5.30	Nikolay Aleksandrov
5.30 - 6.30	`Cocktail Reception` *(Sherrerd Hall Atrium)*
SATURDAY 3/28	
8:00 - 9:00am	`Breakfast`
9:00 - 9:40	Xunyu Zhou
9:40 - 10:20	Vicky Henderson
10:20 - 10:50	`Coffee Break`
10:50 - 11:30	Wei Xiong
11:30 - 12:10	Hanqing Jin
12:10 - 12:30	Joe Yang
12.30 - 12.50	Alok Gupta
12:50 - 2:20pm	`Lunch` *(Sherrerd Hall Atrium)*
2:20 - 3:00	John Mulvey
3:00 - 3:40	Christoph Reisinger
3:40 - 4:10	`Coffee Break`
4:10 - 4:50	Michael Monoyios
4:50 - 5:10	Ke Yu
5.10 - 5.30	Xuedong He

Titles & Abstracts

- René Carmona, (Princeton University)
  Reduced Form Models for Emissions Cap-and-Trade Schemes
  The talk is concerned with the mathematical theory of the cap-and-trade schemes touted as the most efficient way to curb Green House Gas emissions. After quickly reviewing recent results on price formation based on economic equilibrium theory, we propose simple reduced form models which can be calibrated to existing allowance price data, and used to price options on forward allowance prices. This most recent work is motivated by the increasing liquidity of the European Union ETS and the prospect of a unique federal cap-and-trade scheme in the US.

- Paul Glasserman, (Columbia University)
  Risk Horizon and Rebalancing Horizon
  We analyze the impact of portfolio rebalancing frequency on the measurement of risk over a moderately long horizon. This problem arises from an incremental capital charge recently proposed by the Basel Committee on Banking Supervision. The new risk measure calculates VaR over a one-year horizon at a high confidence level and assigns different rebalancing frequencies to different types of assets to capture potential illiquidity. We analyze the difference between discretely and continuously rebalanced portfolios in a simple model of asset dynamics by examining the limit as the rebalancing frequency increases. This leads to alternative approximations at moderate and extreme loss levels. We also show how to incorporate multiple scales of rebalancing frequency in the analysis.

- Vicky Henderson, (Oxford University)
  Prospect Theory, Partial Liquidation and the Disposition Effect
  We solve a liquidation problem for an agent with prospect theory preferences who seeks to sell a portfolio of (divisible) claims on an underlying asset. Our methodology enables us to consider different formulations of prospect preferences in the literature, and various asset price processes. We find that these differences in specification are important - for instance, with piecewise power functions (but not piecewise exponentials) the agent may voluntarily liquidate at a loss relative to break-even. Further, we find that the likelihood of liquidating at a (small) gain is much higher than liquidating at a (large) loss, consistent with the disposition effect documented in empirical and experimental studies. The ability to partially liquidate also has significant consequences. The prospect agent liquidates the entire position at once, in contrast to behavior under standard concave preferences. If the position is divisible, under piecewise exponential functions, the agent no longer liquidates at the break-even level, and even if the asset is very poor, prefers to gamble on the possibility of liquidating at a gain. Finally, the piecewise power specification remains consistent with the disposition effect, albeit where the whole portfolio is sold at once.

- Sam Howison, (Oxford University)
  The continuous-discrete interface by multiple time scales
  I shall describe the use of the method of multiple time scales to (i) derive the Black-Scholes PDE with a continuous dividend yield from the corresponding problem with many frequent dividend payments (ii) do the same for continuously and discretely sampled Asian options. Although the examples are specific the technique is quite general.

- Hanqing Jin, (Oxford University)
  Loss Control in Behavioral Portfolio Selection
  For an unconstrained behavioral portfolio selection in a continuous-time market, the optimal portfolio will suffer some constant loss when the market is bad. The possible loss of a greedy investor may be huge enough to trigger disastrous events, (e.g., bankruptcy). In this work, we restrict the possible loss to be under some safety level, and study the portfolio selection problem with a behavioral criterion with S-shaped value function and probability distortions. Concrete examples will be solved explicitly to show the structure of the optimal portfolio.

  This is a joint work with Xun Yu Zhou and Song Zhang.

- Jakub Jurek, (Princeton University)
  Crash-Neutral Currency Carry Trades
  Currency carry trades implemented within G10 currencies have historically delivered significant excess returns with annualized Sharpe ratios in excess of one. This paper investigates whether these excess returns reflect compensation for exposure to crash risk by analyzing the time-series dynamics of the moments of the risk-neutral distribution extracted from currency options, and by examining returns to crash-neutral currency carry trades in which exposure to crashes has been hedged by combining positions in currencies and currency options. Risk-neutral and realized skewness are shown to move in opposite directions in response to realized currency returns such that insurance against currency crashes is cheapest precisely when it is needed most. Although excess returns to crash-neutral strategies decline relative to their unhedged counterparts, they remain positive and highly statistically significant. The results indicate that crash risk premia can explain 30-40% of the total excess return to currency carry trades. Rationalizing the entirety of the excess return via a crash risk premium would require implied volatilities of out-of-the-money currency options to be roughly four times greater than those observed in the data.

- Dmitry Kramkov, (Carnegie Mellon & Oxford Universities)
  A model for a large investor trading at market indifference prices
  We develop a continuous-time model for a large economic agent where she is trading with market makers at their utility indifference prices. We start with the case of simple strategies, where trades occur at a finite number of times, and then use them to define general investment policies. We discuss the mathematical challenges arising in this passage from discrete to continuous-time trading and present partial solutions. The presentation is based on a joint project with Peter Bank from Technical University of Berlin.

- Michael Monoyios, (Oxford University)
  Asymptotic expansions for optimal hedging of basis risk with partial information
  We analyse the utility-based hedging of a claim on a non-traded asset with a correlated traded asset, in the partial information case when the agent does not know the values of the asset price drifts. The drifts are modelled as random variables whose prior distribution is updated via a Kalman-Bucy filter. This restores a full information model with random drift parameters. The resulting optimal hedging problem is solved with exponential utility. Unlike the full information case, the indifference price depends on the traded stock price as well as on the non-traded asset price underlying the claim, and the optimal hedging formula contains an additional term reflecting the risk induced by parameter uncertainty. A perturbation expansion is developed for the indifference price and associated hedging strategy, valid for small values of risk aversion or for small positions in the claim. Comparisons are drawn with the expansion obtained in the full information case. Numerical experiments show that optimal hedging combined with Bayesian learning is effective in dealing with the risk from market incompleteness and from drift parameter uncertainty.

- John Mulvey, (Princeton University)
  Developing Robust Recommendations for Sector-Based Portfolio Models
  In a previous paper, we showed that sector-based asset categories provide greater diversification potential than traditional style breakouts value/growth and large/mid/small. The advantage is especially pertinent during periods of high turbulence and contagion. This paper extends the research to demonstrate that sector categories can be employed to find robust recommendations for optimal asset allocation. Our model is new approach that focuses on the worst-deciles vis-à-vis historical time periods. The model outperforms traditional benchmarks by protecting the investors capital during loss episodes, across all developed markets including Japan over long time periods.

  Joint work with Woo Chang Kim.

- Jan Obloj, (Oxford University)
  Robust pricing and hedging of double no-touch options and the resulting notions of weak arbitrage
  Double no-touch options, contracts which pay out a fixed amount provided an underlying asset remains within a given interval, are commonly traded, particularly in FX markets. In the talk we present model-free bounds on the price of these options based on the prices of more liquidly traded options (call and digital call options). Key steps are the construction of super- and sub-hedging strategies to establish the bounds, and the use of Skorokhod embedding techniques to show the bounds are the best possible.

  When establishing rigorous bounds, we consider carefully what is meant by arbitrage in settings where there is no *a priori* known probability measure. We discuss two natural extensions of the notion of arbitrage, weak arbitrage and weak free lunch with vanishing risk, which are needed to establish equivalence between the lack of arbitrage and the existence of a market model.

- Christoph Reisinger, (Oxford University)
  Simulation of an SPDE model for a credit basket
  The firm value distribution in a large (credit) portfolio can be approximated by the solution to a stochastic partial differential equation (SPDE). We motivate this limit and use it to compute tranche spreads for basket credit derivatives. A naive Monte Carlo solver on top of a PDE solver is computationally costly, and therefore attention has to be paid to the implementation of default events, boundary conditions, and data smoothing, to ensure fast convergence. The reduced set of effective parameters can be calibrated to the firms' individual credit default swaps and to observed tranche prices for basket products.

- Ramon van Handel, (Princeton University)
  Why are nonlinear filters stable?
  Recent particle filtering algorithms for estimation in partially observed systems exhibit remarkable performance, which is not fully explained by typical convergence results. The investigation of such problems naturally leads to the stability theory of nonlinear filters. Despite much work on this topic, few general results are available.

  I will outline a general stability theory for nonlinear filters which attempts to answer the question: why are nonlinear filters stable? In particular, we will see how both the ergodic properties of the signal and the structure of the observations enter the picture. These results largely settle several old problems in filtering theory, including the resolution of a serious gap in a classic paper by Kunita on this topic. If time permits, I will briefly discuss in more detail how such results can help us understand the performance of particle filtering algorithms.

- Wei Xiong, (Princeton University)
  Dynamic Bank Runs
  We develop a dynamic model of bank runs. A bank finances its long-term investment by rolling over short-term debts with a continuum of creditors, whose contract periods are staggering. In deciding whether to roll over the debt, each creditor faces the future rollover risk of the bank with other creditors, i.e., the bank fundamental could fall during his contract period, causing other maturing creditors to withdraw money and forcing the bank to liquidate its asset prematurely. Different from the static bank-run models with multiple equilibria, we derive a unique monotone equilibrium, in which creditors coordinate their asynchronous rollover actions based on observable fundamental shocks. Our model captures a central element of the ongoing financial crisis–even in the absence of any fundamental deterioration, fluctuations in the capital markets such as small changes in the volatility and liquidation value of the bank asset, because of their roles in determining the bank's future rollover risk, could trigger preemptive runs by creditors on a solvent bank.

- Xunyu Zhou, (Oxford University)
  Greed, Leverage, and Potential Losses: A Prospect Theory Perspective
  It has been largely agreed that human greed is one of the major causes of the current financial crisis. Strong greed in financial investment practice is typically accompanied by an incredibly high level of leverage, leading to catastrophic losses when the market goes wrong. In this work, we quantify the notion of greed, and investigate its connection with leverage and potential losses, in the context of continuous-time behavioural portfolio selection under prospect theory. We show that the asymptotic probability

of ending up with a loss situation is a constant when the greed approaches infinity, yet the leverage level and the magnitude of losses once a loss situation does occur will increase monotonically with respect to the greed, and grow to infinitely large when the greed is infinitely strong. The results in turn suggest a model where the losses are a priori capped.

This is a joint work with Hanqing Jin.

Student Talks

- Nikolay Aleksandrov, (Oxford University)
  A dual approach to multiple exercise option problems under constraints
  This talk is about the pricing of multiple exercise options in discrete time. This type of option can be exercised up to a fixed finite number of times over the lifetime of the contract. We allow multiple exercise of the option at each time point up to a constraint, a feature relevant for pricing swing options in energy markets. It is shown that, in the case where an option can be exercised an equal number of times at each time point, the problem can be reduced to the case of a single exercise possibility at each time. In the general case there is not a solution of this type. We develop a dual representation for the problem and give an algorithm for calculating both lower and upper bounds for the prices of such multiple exercise options.

- Alok Gupta, (Oxford University)
  Optimal Bayesian Hedging Strategies In The Local Volatility Model
  We investigate calibrating the local volatility surface using a rigorous Bayesian framework. By incorporating potential calibration error into our method we design optimal hedges that minimise expected loss statistics based on different Bayesian loss functions decided by an investor's preferences. Comparisons made with the standard hedge strategies show the Bayesian hedges to outperform traditional methods.

- Xuedong He, (Oxford University)
  Portfolio Choice via Quantiles
  A new portfolio choice model in continuous time is formulated for both complete and incomplete markets, where the quantile function of the terminal cash flow, instead of the cash flow itself, is taken as the decision variable. This formulation covers a wide body of existing and new models with law-invariant preference measures, including expected utility maximisation, mean-variance, goal reaching, Yaari's dual model, Lopes' SP/A model, behavioural model under prospect theory, as well as those explicitly involving VaR and CVaR in objectives and/or constraints. A solution scheme to this quantile model is proposed, and then demonstrated by solving analytically the goal-reaching model and Yaari's dual model. A general property derived for the quantile model is that the optimal terminal payment is anti-comonotonic with the pricing kernel (or with the minimal pricing kernel in the case of an incomplete market) if the investment opportunity set is deterministic. As a consequence, the mutual fund theorem still holds in a market where rational and irrational agents co-exist.

- Gechun Liang, (Oxford University)
Backward stochastic dynamics under adapteness constraints
We consider the following backward stochastic dynamics based on a general filtered probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, P)$:

$$\begin{cases} dY_t = -f_0(t, Y_t, L(M)_t)dt - \sum_{i=1}^N f_i(t, Y_t)dB_t^i + dM_t, \\ Y_T = \xi \in \mathcal{F}_T \end{cases}$$

where B is an N-dimensional Brownian motion as given, and M is a square-integrable martingale to be determined. Under *adapteness* constraints on Y, we prove that the equation admits a solution pair (Y, M), which is unique in the sense of strict solutions to be introduced in the main text. The martingale representation is not required, and in order to prove the existence and uniqueness, we establish the existence and uniqueness of a functional differential equation, in a form $V = \mathbb{L}(V)$, where $\mathbb{L}$ is a non-linear functional. Finally we indicate a connection between the backward stochastic dynamics discussed here and a class of non-linear PDE, namely semi-linear PDE with non-local integral term.

This is a joint work with Terry Lyons and Zhongmin Qian.

- Sergey Nadtochiy, (Princeton University)
Static Hedging Under Zero Drift CEV
We consider the problem of static hedging strategy for Barrier Options in the model where underlying is given by driftless CEV process with, possibly, independent time - change. We obtain analytic expression for the payoff of a European-type contingent claim that has the same price as the barrier option before hitting the barrier and study the approximation of the price of such claim with two vanilla options.

- Mitja Stadje, (Princeton University) [CANCELLED]
Extending Time-Consistent Risk Measures From Discrete Time to Continuous Time: a Convergence Approach
The aim of this talk is to present an approach for the transition from risk measures in discrete time to their counterparts in continuous time. It is shown that a large class of risk measures in continuous time can be obtained very naturally as limits of stable time-consistent risk measures in a discrete setting. The discrete-time risk measures are constructed from properly rescaled ('tilted') one-period risk measures, using a d-dimensional random walk converging to a Brownian Motion. Under suitable conditions (covering the classical one-period risk measures) we obtain convergence of the discrete risk measures to the solution of a backward stochastic differential equation, defining a risk measure in continuous time, whose driver can then be viewed as the continuous-time analogue of the discrete 'driver' characterizing the one-period risk. We derive the limiting drivers for the semi-deviation risk measure, Average Value at Risk and the Gini risk measure in closed form. This is joint work with my PhD advisor Patrick Cheridito.

- Ke Yu, (Princeton University)
  Asset Allocation with Gross Exposure Constraints for Vast Portfolios Utilizing High Frequency Data

  Under Markowitz's mean-variance asset allocation framework, the estimation of the covariance matrix is one of the crucial problems to be dealt with. Fan, Zhang and Yu showed that with gross-exposure constraint the empirically selected optimal portfolios based on estimated covariance matrices have similar performance to the theoretical optimal portfolios. We are exploring the ways of estimating covariance matrices utilizing rich high frequency trading data such as all-refresh-time covariance matrix estimation and pairwise-refresh-time covariance matrix estimation. We are also comparing our approaches with the low frequency approaches of Fan, Zhang and Yu. The theoretical results are accompanied and supported by the simulation and empirical studies on the 30 component stocks of Dow Jones Industrial Average.

- Forrest Zhang, (Princeton University)
  Asset Allocation with Gross Exposure Constraints for Vast Portfolios
  Markowitz (1952, 1959) laid down the ground-breaking work on the mean-variance analysis. Under his framework, the theoretical optimal allocation vector can be very different from the estimated one for large portfolios due to the intrinsic difficulty of estimating a vast covariance matrix and return vector. This can result in adverse performance in portfolio selected based on empirical data due to the accumulation of estimation errors. We address this problem by introducing the gross-exposure constrained mean-variance portfolio selection. We show that with gross-exposure constraint the empirically selected optimal portfolios based on estimated covariance matrices have similar performance to the theoretical optimal portfolios and there is no error accumulation effect from estimation of vast covariance matrices. This gives theoretical justification to the empirical results in Jagannathan and Ma (2003). We also show that the no-short-sale portfolio is not diversified enough and can be improved by allowing some short positions. As the constraint on short sales relaxes, the number of selected assets gradually increases and finally reaches the total number of stocks when tracking portfolios or selecting assets. This achieves the optimal sparse portfolio selection, which has close performance to the theoretical optimal one. Among 1000 stocks, for example, we are able to identify all optimal subsets of portfolios of different sizes, their associated allocation vectors, and their estimated risks. The utility of our new approach is illustrated by simulation and empirical studies on the 100 Fama-French industrial portfolios and the 600 stocks randomly selected from Russell 3000.

  Joint work with Jianqing Fan and Ke Yu.

- Z. Joseph Yang, (Princeton University)
  Volatility and Illiquidity Trading
  We focus on the market of a certain illiquid asset and the strategic play between oligopolistic players in continuous-time. First, we provide a mathematical account for the reasonableness of the formulation of the permanent and temporary components of market impact under the time-homogeneity assumption. In addition to previous researchers, we allow each player to use closed-loop strategies (strategies with feedbacks in real-time) and thus ushered in volatility as a relevant factor to this story. We formulate the Nash-equilibrium of this illiquidity trading game by a system of

HJB equations, and point out that the indispensible way to analyze numerically the Nash-equilibrium resorts to the incorporation of a little viscosity.

Joint work with René Carmona.

Organizers

Ronnie Sircar (Princeton University)
Vicky Henderson (Oxford-Man Institute)

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