Greed, Leverage, and Potential Losses: A Prospect Theory Perspective

Xunyu Zhou

Based on joint work with Hanqing Jin

28th March 2009/Oxford–Princeton Workshop
1 Prologue

2 Behavioural Portfolio Choice under Prospect Theory

3 Greed, Leverage and Potential Losses

4 Epilogue
Prologue: Financial Crisis Blame Game

Who’s to blame for this financial crisis: The top 10 list
Prologue: Financial Crisis Blame Game

Who’s to blame for this financial crisis: The top 10 list

10 Ratings Agencies
Prologue: Financial Crisis Blame Game

Who’s to blame for this financial crisis: The top 10 list

10 Ratings Agencies
9 Alan Greenspan
Prologue: Financial Crisis Blame Game

Who’s to blame for this financial crisis: The top 10 list

10 Ratings Agencies
9 Alan Greenspan
8 George W. Bush
Prologue: Financial Crisis Blame Game

Who’s to blame for this financial crisis: The top 10 list

10 Ratings Agencies
9 Alan Greenspan
8 George W. Bush
7 Wen Jiabao
Prologue: Financial Crisis Blame Game

Who’s to blame for this financial crisis: The top 10 list

10  Ratings Agencies
9   Alan Greenspan
8   George W. Bush
7   Wen Jiabao
6   Bernard Madoff
Prologue: Financial Crisis Blame Game

Who’s to blame for this financial crisis: The top 10 list

10  Ratings Agencies
9   Alan Greenspan
8   George W. Bush
7   Wen Jiabao
6   Bernard Madoff
5   Americans
Prologue: Financial Crisis Blame Game

Who’s to blame for this financial crisis: The top 10 list

10 Ratings Agencies
9 Alan Greenspan
8 George W. Bush
7 Wen Jiabao
6 Bernard Madoff
5 Americans
4 Chinese
Prologue: Financial Crisis Blame Game

Who’s to blame for this financial crisis: The top 10 list

10 Ratings Agencies
9 Alan Greenspan
8 George W. Bush
7 Wen Jiabao
6 Bernard Madoff
5 Americans
4 Chinese
3 Economists
Prologue: Financial Crisis Blame Game

Who’s to blame for this financial crisis: The top 10 list

10  Ratings Agencies
9   Alan Greenspan
8   George W. Bush
7   Wen Jiabao
6   Bernard Madoff
5   Americans
4   Chinese
3   Economists
2   Quants
Prologue: Financial Crisis Blame Game

Who’s to blame for this financial crisis: The top 10 list

10 Ratings Agencies
9 Alan Greenspan
8 George W. Bush
7 Wen Jiabao
6 Bernard Madoff
5 Americans
4 Chinese
3 Economists
2 Quants
1 Financial Engineers and Financial Mathematicians (i.e. you and me)
Ultimate Culprits

- “Greed, Stupidity, Delusion – and Some More Greed” (John Steele Gordon; NY Times, 23 March 2009)
- “Rowan Williams says ’human greed’ to blame for financial crisis” (Times Online, 15 October 2008)
- German finance minister Peer Steinbrück denounces US greed (October 2008)
- “...hardwired human behavior coupled with free enterprise and modern capitalism” (Andrew Lo)
Goal of This Work

- Quantify “greed”, and define “leverage” and “potential losses” in the context of behavioural portfolio choice.
Goal of This Work

- Quantify “greed”, and define “leverage” and “potential losses” in the context of behavioural portfolio choice
- Explore connection amongst the three via post-optimality/sensitivity analyses
Goal of This Work

- Quantify “greed”, and define “leverage” and “potential losses” in the context of behavioural portfolio choice
- Explore connection amongst the three via post-optimality/sensitivity analyses
- Suggest alternative models where greed is contained (if indirectly)
The Market

- Continuous time
The Market

- Continuous time
- Tame portfolios
The Market

- Continuous time
- Tame portfolios
- Arbitrage-free and complete market
Expected Utility Maximization

\[
\begin{align*}
\text{Max} & \quad Eu(X) \\
\text{Subject to} & \quad E[\rho X] = x_0, \\
& \quad X \geq 0
\end{align*}
\]
Expected Utility Maximization

\[
\begin{align*}
\max_{X} & \quad E u(X) \\
\text{Subject to} & \quad E[\rho X] = x_0, \\
& \quad X \geq 0
\end{align*}
\]

where

- \(X\): terminal payoff (cash flow) – an \(\mathcal{F}_T\) random variable
Expected Utility Maximization

\[
\begin{align*}
\max_X & \quad Eu(X) \\
\text{Subject to} & \quad E[\rho X] = x_0, \\
& \quad X \geq 0
\end{align*}
\]

where

- \( X \): terminal payoff (cash flow) – an \( \mathcal{F}_T \) random variable
- \( u(\cdot) \): utility function
Expected Utility Maximization

\[
\begin{align*}
\max_X & \quad Eu(X) \\
\text{Subject to} & \quad E[\rho X] = x_0, \\
& \quad X \geq 0
\end{align*}
\]

where

- \(X\): terminal payoff (cash flow) – an \(\mathcal{F}_T\) random variable
- \(u(\cdot)\): utility function
- \(\rho\): pricing kernel – an \(\mathcal{F}_T\) random variable
Expected Utility Maximization

$$\begin{align*}
\text{Max}_{X} & \quad Eu(X) \\
\text{Subject to} & \quad E[\rho X] = x_0, \\
& \quad X \geq 0
\end{align*}$$

where

- $X$: terminal payoff (cash flow) – an $\mathcal{F}_T$ random variable
- $u(\cdot)$: utility function
- $\rho$: pricing kernel – an $\mathcal{F}_T$ random variable
- $x_0$: initial wealth
Expected Utility Maximization

\[
\begin{align*}
\max_{X} & \quad Eu(X) \\
\text{Subject to} & \quad E[\rho X] = x_0, \\
& \quad X \geq 0
\end{align*}
\]

where

- \( X \): terminal payoff (cash flow) – an \( \mathcal{F}_T \) random variable
- \( u(\cdot) \): utility function
- \( \rho \): pricing kernel – an \( \mathcal{F}_T \) random variable
- \( x_0 \): initial wealth

Merton (1971); abundant research thereafter
Human Judgement Implied by Expected Utility Theory

- Expected Utility Theory (EUT): Dominant model for decision making under uncertainty, including financial asset allocation
Human Judgement Implied by Expected Utility Theory

- Expected Utility Theory (EUT): Dominant model for decision making under uncertainty, including financial asset allocation
- Basic tenets of human judgement implied by EUT in the context of asset allocation:
Human Judgement Implied by Expected Utility Theory

- Expected Utility Theory (EUT): Dominant model for decision making under uncertainty, including financial asset allocation
- Basic tenets of human judgement implied by EUT in the context of asset allocation:
  - **Source of satisfaction**: Investors evaluate assets according to final asset positions
Human Judgement Implied by Expected Utility Theory

- Expected Utility Theory (EUT): Dominant model for decision making under uncertainty, including financial asset allocation
- Basic tenets of human judgement implied by EUT in the context of asset allocation:
  - Source of satisfaction: Investors evaluate assets according to final asset positions
  - Attitude towards risk: Investors are always risk averse (concave utility)
Human Judgement Implied by Expected Utility Theory

- Expected Utility Theory (EUT): Dominant model for decision making under uncertainty, including financial asset allocation
- Basic tenets of human judgement implied by EUT in the context of asset allocation:
  - **Source of satisfaction**: Investors evaluate assets according to final asset positions
  - **Attitude towards risk**: Investors are always risk averse (concave utility)
  - **Beliefs about future**: Investors are able to objectively evaluate probabilities of future returns
Neoclassical Economics

- Neoclassical economics: Economics, albeit primarily about human activities, can be made as logical, precise and predictable as natural sciences.
Neoclassical Economics

- Neoclassical economics: Economics, albeit primarily about human activities, can be made as logical, precise and predictable as natural sciences
- Underlying assumptions:
Neoclassical Economics

- Neoclassical economics: Economics, albeit primarily about human activities, can be made as logical, precise and predictable as natural sciences.
- Underlying assumptions:
  - People have rational preferences among outcomes.
Neoclassical economics: Economics, albeit primarily about human activities, can be made as logical, precise and predictable as natural sciences.

Underlying assumptions:
- People have rational preferences among outcomes
- Individuals maximise utility and firms maximise profits
Neoclassical Economics

- Neoclassical economics: Economics, albeit primarily about human activities, can be made as logical, precise and predictable as natural sciences

- Underlying assumptions:
  - People have rational preferences among outcomes
  - Individuals maximise utility and firms maximise profits
  - People act independently on the basis of full and relevant information
Market Is Always Right

- Efficient market hypothesis (Eugene Fama 1960s): Financial markets “informationally efficient”, or “prices are right”
Market Is Always Right

- Efficient market hypothesis (Eugene Fama 1960s): Financial markets “informationally efficient”, or “prices are right”
- Chicago school (Milton Friedman 1912-2006): regulation and other government intervention always inefficient compared to a free market
Efficient market hypothesis (Eugene Fama 1960s): Financial markets “informationally efficient”, or “prices are right”

Chicago school (Milton Friedman 1912-2006): regulation and other government intervention always inefficient compared to a free market

Reaganomics: “Only by reducing the growth of government, can we increase the growth of the economy”
Sense and Sensibility

Is human judgement always rational (and therefore market is always right)?
Sense and Sensibility

- Is human judgement always rational (and therefore market is always right)?
- Substantial evidences suggest systematic violation of EUT
Is human judgement always rational (and therefore market is always right)?

Substantial evidences suggest systematic violation of EUT

- *Source of satisfaction*: Investors evaluate assets according to deviation from a reference point
Is human judgement always rational (and therefore market is always right)?

Substantial evidences suggest systematic violation of EUT

- **Source of satisfaction**: Investors evaluate assets according to deviation from a reference point
- **Attitude towards risk**: Investors are **not** globally risk averse, and distinctively more sensitive to losses than to gains
Is human judgement always rational (and therefore market is always right)?

Substantial evidences suggest systematic violation of EUT

- **Source of satisfaction**: Investors evaluate assets according to deviation from a reference point.
- **Attitude towards risk**: Investors are not globally risk averse, and distinctively more sensitive to losses than to gains.
- **Beliefs about future**: Investors exaggerate small probabilities.
An Experiment on Comparison

- **Experiment (Alan Greenspan):** compare the following two job offers:
  - **A:** Earn £120,000/year while all your colleagues earn at least £240,000/year
  - **B:** Earn £110,000/year while all your colleagues earn at most £55,000/year
Experiment (Alan Greenspan): compare the following two job offers:

A: Earn £120,000/year while all your colleagues earn at least £240,000/year
B: Earn £110,000/year while all your colleagues earn at most £55,000/year

B was more popular
An Experiment on Comparison

- Experiment (Alan Greenspan): compare the following two job offers:
  - A: Earn £120,000/year while all your colleagues earn at least £240,000/year
  - B: Earn £110,000/year while all your colleagues earn at most £55,000/year
- B was more popular
- People are born to compare
The AIG Saga

Why are people so furious about AIG bonuses ($218m) saga?
The AIG Saga

Why are people so furious about AIG bonuses ($218m) saga?

After all, it’s a small amount compared with

- $170b government rescue money
- $96b paid-out to CDSs and security-lending counterparties
The AIG Saga

Why are people so furious about AIG bonuses ($218m) saga?

After all, it’s a small amount compared with

- $170b government rescue money
- $96b paid-out to CDSs and security-lending counterparties

The answer is behavioural (yes people are irrational and they compare)!
Prospect Theory

- Reference point (Kahneman and Tversky 1979) or customary wealth (Markowitz 1952) that defines gains and losses
Prospect Theory

- **Reference point** (Kahneman and Tversky 1979) or *customary wealth* (Markowitz 1952) that defines gains and losses
- *S*-shaped utility function (risk-averse on gains, risk-seeking on losses), steeper on losses than on gains
Prospect Theory

- *Reference point* (Kahneman and Tversky 1979) or *customary wealth* (Markowitz 1952) that defines gains and losses
- S-shaped utility function (risk-averse on gains, risk-seeking on losses), steeper on losses than on gains
- Probability distortions
$S$-shaped Function
Probability Distortion Function

\[ T(p) \]

\[ 0 \leq p \leq 1 \]
KT’s Utility and Distortions

Kahneman and Tversky (1992) suggest the following
KT’s Utility and Distortions

Kahneman and Tversky (1992) suggest the following

- Utility function

\[ u(x) = \begin{cases} 
  x^\alpha, & x \geq 0, \\
  -k(-x)^\beta, & x < 0 
\end{cases} \]

where \( \alpha = \beta = 0.88, \ k = 2.25 \)
KT’s Utility and Distortions

Kahneman and Tversky (1992) suggest the following

- Utility function

\[ u(x) = \begin{cases} 
  x^\alpha, & x \geq 0, \\
  -k(-x)^\beta, & x < 0 
\end{cases} \]

where \( \alpha = \beta = 0.88, \ k = 2.25 \)

- Probability distortion functions

\[ T_+(p) = \frac{p^\gamma}{(p^\gamma+(1-p)^\gamma)^{1/\gamma}} \]

\[ T_-(p) = \frac{p^\delta}{(p^\delta+(1-p)^\delta)^{1/\delta}} \]

where \( \gamma = 0.61, \ \delta = 0.69 \)
Behavioural Portfolio Choice à la Prospect Theory

\[
\begin{align*}
\max_{X} & \quad \int_{0}^{\infty} T_+ (P (u_+ ((X - B)_+) > x)) \, dx \\
& \quad - \int_{0}^{\infty} T_- (P (u_- ((X - B)_-) > x)) \, dx \\
\text{Subject to} & \quad E[\rho X] = x_0
\end{align*}
\]
Max_X \int_0^\infty T_+ (P(u_+ ((X - B)_+) > x)) \, dx \\
- \int_0^\infty T_- (P(u_- ((X - B)_-) > x)) \, dx \\
Subject to \quad E[\rho X] = x_0 \\

where 

- **B**: reference point in wealth (possibly random)
Behavioural Portfolio Choice à la Prospect Theory

\[
\begin{align*}
\text{Max}_{X} & \quad \int_{0}^{\infty} T_+ (P (u_+ ((X - B)_+) > x)) \, dx \\
& \quad - \int_{0}^{\infty} T_- (P (u_- ((X - B)_-) > x)) \, dx \\
\text{Subject to} & \quad E[\rho X] = x_0
\end{align*}
\]

where

- \( B \): reference point in wealth (possibly random)
- \( X \): terminal payoff
Max_{X} \int_{0}^{\infty} T_{+} (P (u_{+} ((X - B)_{+}) > x)) \, dx \\
- \int_{0}^{\infty} T_{-} (P (u_{-} ((X - B)_{-}) > x)) \, dx \\
Subject to \quad E[\rho X] = x_{0} \\

where

- \textbf{B}: reference point in wealth (possibly random) \\
- \textbf{X}: terminal payoff \\
- T_{\pm} : [0, 1] \rightarrow [0, 1] \text{ probability distortions}
Max \( \int_0^\infty T_+ (P (u_+ ((X - B)_+) > x)) \) \( dx \)
\[ - \int_0^\infty T_- (P (u_- ((X - B)_-) > x)) \) \( dx \)
Subject to \( E[\rho X] = x_0 \)

where

- \( B \): reference point in wealth (possibly random)
- \( X \): terminal payoff
- \( T_\pm : [0, 1] \rightarrow [0, 1] \) probability distortions
- \( u_+(x)1_{x \geq 0} - u_-(x)1_{x < 0} \): overall utility function
Behavioural Portfolio Choice à la Prospect Theory

Max
\[
\int_0^\infty T_+ (P (u_+ ((X - B)_+) > x)) dx
- \int_0^\infty T_- (P (u_- ((X - B)_-) > x)) dx
\]

Subject to \( E[\rho X] = x_0 \)

where

- \( B \): reference point in wealth (possibly random)
- \( X \): terminal payoff
- \( T_\pm : [0, 1] \rightarrow [0, 1] \) probability distortions
- \( u_+(x)\mathbf{1}_{x\geq 0} - u_-(x)\mathbf{1}_{x<0} \): overall utility function
- \( \rho \): pricing kernel with CDF \( F(\cdot) \)
Behavioural Portfolio Choice à la Prospect Theory

\[
\begin{align*}
\max_X \int_0^\infty & \left( T_+ (P (u_+ ((X - B)_+) > x)) - \int_0^\infty T_- (P (u_- ((X - B)_-) > x)) \right) dx \\
\text{Subject to} \quad & E[\rho X] = x_0
\end{align*}
\]

where

- \( B \): reference point in wealth (possibly random)
- \( X \): terminal payoff
- \( T_\pm : [0, 1] \to [0, 1] \) probability distortions
- \( u_+ (x) 1_{x \geq 0} - u_- (x) 1_{x < 0} \): overall utility function
- \( \rho \): pricing kernel with CDF \( F(\cdot) \)
- \( x_0 \): initial budget
Max \( \int_{0}^{\infty} T_+ (P (u_+ ((X - B)_+) > x)) \, dx \)

\(- \int_{0}^{\infty} T_- (P (u_- ((X - B)_-) > x)) \, dx \)

Subject to \( E[\rho X] = x_0 \)

where

- \( B \): reference point in wealth (possibly random)
- \( X \): terminal payoff
- \( T_{\pm} : [0, 1] \rightarrow [0, 1] \) probability distortions
- \( u_+ (x) 1_{x \geq 0} - u_- (x) 1_{x < 0} \): overall utility function
- \( \rho \): pricing kernel with CDF \( F(\cdot) \)
- \( x_0 \): initial budget

Approaches

- Expected utility: stochastic control/HJB, martingale/convex duality
Approaches

- Expected utility: stochastic control/HJB, martingale/convex duality
- Prospect model: ???
Approaches

- Expected utility: stochastic control/HJB, martingale/convex duality
- Prospect model: ???
  - Nonconcave in $X$: convex duality fails
Approaches

- Expected utility: stochastic control/HJB, martingale/convex duality
- Prospect model: ???
  - Nonconcave in $X$: convex duality fails
  - Nonlinear expectation with Choquet integration: time-consistency or HJB fails
**Jin and Zhou’s Solution**

**Assumption.** \( u_-(\cdot) \) strictly concave at 0; \( F^{-1}(z)/T'_+(z) \) non-decreasing in \( z \in (0, 1] \); \( \lim \inf_{x \to +\infty} \left( \frac{-x u''_+(x)}{u'_+(x)} \right) > 0 \);

\[ E \left[ u_+ \left( (u'_+)^{-1} \left( \frac{\rho}{T'_+(F(\rho))} \right) \right) T'_+(F(\rho)) \right] < +\infty \]
Jin and Zhou’s Solution

**Assumption.** $u_-(\cdot)$ strictly concave at 0; $F^{-1}(z)/T'_+(z)$ non-decreasing in $z \in (0, 1]$; \( \lim \inf_{x \to +\infty} \left( \frac{-xu''_+(x)}{u'_+(x)} \right) > 0; \)

\[
E \left[ u_+ \left( (u'_+)^{-1} \left( \frac{\rho}{T'_+(F(\rho))} \right) \right) T'_+(F(\rho)) \right] < +\infty
\]

Consider a mathematical programme in $(c, x_+)$:

Maximise
\[
E \left[ u_+ \left( (u'_+)^{-1} \left( \frac{\lambda(c, x_+)\rho}{T'_+(F(\rho))} \right) \right) T'_+(F(\rho))1_{\rho \leq c} \right] \\
-u_-(\frac{x_+-(x_0-E[\rho B])}{E[\rho 1_{\rho > c}]})T_-(1 - F(c))
\]

subject to \begin{align*}
\rho &\leq c \leq \bar{\rho}, \quad x_+ \geq (x_0 - E[\rho B])^+, \\
x_+ & = 0 \text{ when } c = \rho, \quad x_+ = x_0 - E[\rho B] \text{ when } c = \bar{\rho},
\end{align*}

where $\lambda(c, x_+)$ satisfies \[
E \left[ (u'_+)^{-1} \left( \frac{\lambda(c, x_+)\rho}{T'_+(F(\rho))} \right) \rho 1_{\rho \leq c} \right] = x_+
\]
Jin and Zhou’s Solution

Assumption. \( u_- (\cdot) \) strictly concave at 0; \( F^{-1}(z) / T'_+(z) \) non-decreasing in \( z \in (0, 1] \); \( \lim \inf_{x \to +\infty} \left( -\frac{u''_+(x)}{u'_+(x)} \right) > 0 \);

\[
E \left[ u_+ \left( \left( u'_+ \right)^{-1} \left( \frac{\rho}{T'_+(F(\rho))} \right) \right) T'_+(F(\rho)) \right] < +\infty
\]

Consider a mathematical programme in \((c, x_+)\):

Maximise

\[
E \left[ u_+ \left( \left( u'_+ \right)^{-1} \left( \frac{\lambda(c, x_+) \rho}{T'_+(F(\rho))} \right) \right) T'_+(F(\rho))1_{\rho \leq c} \right] - u_- \left( \frac{x_+ - (x_0 - E[\rho B])}{E[\rho 1_{\rho > c}]} \right) T_- (1 - F(c))
\]

subject to

\[
\left\{ \begin{array}{l}
\rho \leq c \leq \bar{\rho}, \quad x_+ \geq (x_0 - E[\rho B])^+, \\
x_+ = 0 \text{ when } c = \rho, \quad x_+ = x_0 - E[\rho B] \text{ when } c = \bar{\rho},
\end{array} \right.
\]

where \( \lambda(c, x_+) \) satisfies

\[
E \left[ (u'_+)^{-1} \left( \frac{\lambda(c, x_+) \rho}{T'_+(F(\rho))} \right) \rho 1_{\rho \leq c} \right] = x_+
\]

Optimal solution (Jin and Zhou 2008)

\[
X^* = \left[ \left( u'_+ \right)^{-1} \left( \frac{\lambda \rho}{T'_+(F(\rho))} \right) + B \right] 1_{\rho \leq c^*} - \left[ \frac{x^*_+ - (x_0 - E[\rho B])}{E[\rho 1_{\rho > c^*}]} - B \right] 1_{\rho > c^*}
\]
Future world divided by two states: “Good” or “bad”, completely determined by whether $\rho \leq c^*$ or $\rho > c^*$.
Interpretations and Implications

- Future world divided by two states: “Good” or “bad”, completely determined by whether $\rho \leq c^*$ or $\rho > c^*$
- Gain or loss correspond to good and bad states, respectively
Interpretations and Implications

- Future world divided by two states: “Good” or “bad”, completely determined by whether $\rho \leq c^*$ or $\rho > c^*$
- Gain or loss correspond to good and bad states, respectively
- Optimal strategy is a *gambling* policy, betting on the good state while accepting a loss on the bad
Interpretations and Implications

- Future world divided by two states: “Good” or “bad”, completely determined by whether $\rho \leq c^*$ or $\rho > c^*$
- Gain or loss correspond to good and bad states, respectively
- Optimal strategy is a *gambling* policy, betting on the good state while accepting a loss on the bad
- Everyone gambled before the crisis: **Good state** – US housing market will never fall ... “banks bet heavily on the idea that housing prices at the levels of the middle of 2006 actually made sense” – Paul Krugman; **bad state** – US housing market will fall ...
Interpretations and Implications

- Future world divided by two states: “Good” or “bad”, *completely* determined by whether $\rho \leq c^*$ or $\rho > c^*$
- Gain or loss correspond to good and bad states, respectively
- Optimal strategy is a *gambling* policy, betting on the good state while accepting a loss on the bad
- Everyone gambled before the crisis: **Good state** – US housing market will never fall ... “banks bet heavily on the idea that housing prices at the levels of the middle of 2006 actually made sense” – Paul Krugman; **bad state** – US housing market will fall ... *are you kidding?* (Joseph Cassano)
Interpretations and Implications

- Future world divided by two states: “Good” or “bad”, completely determined by whether $\rho \leq c^*$ or $\rho > c^*$
- Gain or loss correspond to good and bad states, respectively
- Optimal strategy is a *gambling* policy, betting on the good state while accepting a loss on the bad
- Everyone gambled before the crisis: **Good state** – US housing market will never fall ... “banks bet heavily on the idea that housing prices at the levels of the middle of 2006 actually made sense” – Paul Krugman; **bad state** – US housing market will fall ... *are you kidding?* (Joseph Cassano)
- The strategy typically entails a leverage on stocks if the agent starts with a *loss* situation (due, e.g., to high aspiration, such as Jérôme Kerviel or Nick Leeson)
Interpretations and Implications

- Future world divided by two states: “Good” or “bad”, completely determined by whether $\rho \leq c^*$ or $\rho > c^*$
- Gain or loss correspond to good and bad states, respectively
- Optimal strategy is a *gambling* policy, betting on the good state while accepting a loss on the bad
- Everyone gambled before the crisis: **Good state** – US housing market will never fall ... “banks bet heavily on the idea that housing prices at the levels of the middle of 2006 actually made sense” – Paul Krugman; **bad state** – US housing market will fall ... *are you kidding?* (Joseph Cassano)
- The strategy typically entails a leverage on stocks if the agent starts with a *loss* situation (due, e.g., to high aspiration, such as Jérôme Kerviel or Nick Leeson)
- Magnitude of potential losses *dependent of $B$*
Defining Greed

- $x_0$: initial endowment
Defining Greed

- $x_0$: initial endowment
- $B$: reference point (possible random)
Defining Greed

- $x_0$: initial endowment
- $B$: reference point (possible random)
- $B$ changes agent risk attitude fundamentally: so long as $x_0 < E[\rho B]$ the agent is risk-seeking and aggressive
Defining Greed

- $x_0$: initial endowment
- $B$: reference point (possible random)
- $B$ changes agent risk attitude fundamentally: so long as $x_0 < E[\rho B]$ the agent is risk-seeking and aggressive
- greed becomes relevant and significant only when $x_0 < E[\rho B]$
Defining Greed

- $x_0$: initial endowment
- $B$: reference point (possible random)
- $B$ changes agent risk attitude fundamentally: so long as $x_0 < E[\rho B]$ the agent is risk-seeking and aggressive
- Greed becomes relevant and significant only when $x_0 < E[\rho B]$
- The higher the reference point the more likely the agent is to be a risk-taker
Defining Greed

- \( x_0 \): initial endowment
- \( B \): reference point (possible random)
- \( B \) changes agent risk attitude fundamentally: so long as \( x_0 < E[\rho B] \) the agent is risk-seeking and aggressive
- greed becomes relevant and significant only when \( x_0 < E[\rho B] \)
- The higher the reference point the more likely the agent is to be a risk-taker
- a natural definition is the ratio between what the agent is desperate to achieve and what she has to start with

\[
G = \frac{E[\rho B]}{x_0}
\]
Defining Greed

- $x_0$: initial endowment
- $B$: reference point (possible random)
- $B$ changes agent risk attitude fundamentally: so long as $x_0 < E[\rho B]$ the agent is risk-seeking and aggressive
- Greed becomes relevant and significant only when $x_0 < E[\rho B]$
- The higher the reference point the more likely the agent is to be a risk-taker
- A natural definition is the ratio between what the agent is desperate to achieve and what she has to start with

\[
G = \frac{E[\rho B]}{x_0}
\]

- Definition does not work for expected utility model
Defining Leverage

- Leverage: the ratio between borrowing amount and equity in a venture
Defining Leverage

- Leverage: the ratio between borrowing amount and equity in a venture
- Example: you buy a house of £500K, put 10% downpayment and borrow £450K from lender

\[ 50K = 500K - 450K \]

so leverage = \( \frac{450}{50} = 9 \)
Defining Leverage

- Leverage: the ratio between borrowing amount and equity in a venture
- Example: you buy a house of £500K, put 10% downpayment and borrow £450K from lender

\[ 50K = 500K - 450K \]

so leverage = \( \frac{450}{50} = 9 \)

- In the present context agent needs to borrow money to fund her portfolios
Defining Leverage

- Leverage: the ratio between borrowing amount and equity in a venture
- Example: you buy a house of £500K, put 10% downpayment and borrow £450K from lender

\[ 50K = 500K - 450K \]

so leverage = \( \frac{450}{50} = 9 \)

- In the present context agent needs to borrow money to fund her portfolios
- Leverage of any given portfolio: ratio between the \( t = 0 \) value of the borrowing amount and the initial endowment \( x_0 \)
Defining Leverage

- Leverage: the ratio between borrowing amount and equity in a venture
- Example: you buy a house of £500K, put 10% downpayment and borrow £450K from lender

\[ 50K = 500K - 450K \]

so leverage = \( 450/50 = 9 \)

- In the present context agent needs to borrow money to fund her portfolios
- Leverage of any given portfolio: ratio between the \( t = 0 \) value of the borrowing amount and the initial endowment \( x_0 \)

Let \( X \) be terminal wealth of given portfolio starting from \( x_0 \)

\[ X \equiv ((X - B)^+ + B) 1_{X \geq B} - ((X - B)^- - B) 1_{X < B} := X_g - X_l. \]
Defining Leverage

- Leverage: the ratio between borrowing amount and equity in a venture
- Example: you buy a house of £500K, put 10% downpayment and borrow £450K from lender

\[ 50k = 500k - 450k \]

so leverage = \( 450/50 = 9 \)

- In the present context agent needs to borrow money to fund her portfolios

- Leverage of any given portfolio: ratio between the \( t = 0 \) value of the borrowing amount and the initial endowment \( x_0 \)
- Let \( X \) be terminal wealth of given portfolio starting from \( x_0 \)

\[ X \equiv ((X - B)^+ + B) 1_{X \geq B} - ((X - B)^- - B) 1_{X < B} := X_g - X_l. \]

- Agent short sells \( X_l \) to fund long position \( X_g \).
Defining Leverage

- Leverage: the ratio between borrowing amount and equity in a venture
- Example: you buy a house of £500K, put 10% downpayment and borrow £450K from lender

\[ 50K = 500K - 450K \]

so leverage \( \frac{450}{50} = 9 \)
- In the present context agent needs to borrow money to fund her portfolios
- Leverage of any given portfolio: ratio between the \( t = 0 \) value of the borrowing amount and the initial endowment \( x_0 \)
- Let \( X \) be terminal wealth of given portfolio starting from \( x_0 \)

\[ X \equiv ((X - B)^+ + B) 1_{X \geq B} - ((X - B)^- - B) 1_{X < B} := X_g - X_l. \]

- Agent short sells \( X_l \) to fund long position \( X_g \).
- \( L := \frac{E(\rho X_l)}{x_0} \)
Defining Potential Losses

- Potential loss (rate): ratio between the $t = 0$ value of losses and $x_0$, given that losses have occurred
Defining Potential Losses

- Potential loss (rate): ratio between the $t = 0$ value of losses and $x_0$, given that losses have occurred

- $l := E \left( \frac{\rho X_l}{x_0} \middle| X < B \right)$
Hereafter we consider $\log \rho \sim N(\mu, \sigma^2)$ with $\sigma > 0$ and

$$u_+(x) = x^\alpha, \quad u_-(x) = k_- x^\beta, \quad x \geq 0$$

where $k_- > 0$ (loss aversion coefficient) and $0 < \alpha \leq \beta < 1$ are constants.
Hereafter we consider $\log \rho \sim N(\mu, \sigma^2)$ with $\sigma > 0$ and

$$u_+(x) = x^\alpha, u_-(x) = k_- x^\beta, \ x \geq 0$$

where $k_- > 0$ (loss aversion coefficient) and $0 < \alpha \leq \beta < 1$ are constants.

Denote

$$\varphi(c) := E \left[ \left( \frac{T'_+ (F(\rho))}{\rho} \right)^{1/(1-\alpha)} \rho 1_{\rho \leq c} \right] \geq 0, \ 0 \leq c \leq +\infty.$$ 

$$k(c) := \frac{k_- T_- (1 - F(c))}{\varphi(c)^{1-\alpha} (E[\rho 1_{\rho > c}])^\beta} > 0, \ c > 0.$$
Case $\alpha = \beta$: Optimal Terminal Wealth

**Theorem.** (Jin and Zhou 2008) If $\alpha = \beta$ and $x_0 < E[\rho B]$, then behavioural portfolio selection problem has a finite optimal portfolio if and only if $\inf_{c > 0} k(c) > 1$ and

$$\argmin_{c \geq 0} \left[ \left( \frac{k(T - (1 - F(c)))}{(E[\rho 1_{\rho > c}])^\alpha} \right)^{1/(1-\alpha)} - \varphi(c) 1_{c > 0} \right] \neq \emptyset. \quad (1)$$

Moreover, if $c^* > 0$ is one of the minimizers in (1), then optimal terminal wealth is

$$X^* = \frac{x_+}{\varphi(c^*)} \left( \frac{T'(F(\rho))}{\rho} \right)^{1/(1-\alpha)} 1_{\rho \leq c^*} - \frac{x_+ - (x_0 - E[\rho B])}{E[\rho 1_{\rho > c^*}]} 1_{\rho > c^*} + B,$$

where $x_+ := \frac{-(x_0 - E[\rho B])}{k(c^*)^{1/(1-\alpha)} - 1}$; and if $c^* = 0$ is the unique minimizer in (1), then optimal terminal wealth is $X^* = \frac{x_0 - E[\rho B]}{E\rho} + B$. 
Case $\alpha = \beta$: Leverage and Greed

Assume that $x_0 < E(\rho B)$ and that $c^* > 0$ is one of the minimizers in (1)

$$X_l^* = \left( \frac{x^*-(x_0-E[\rho B])}{E[\rho 1_{\rho>c^*}]} - B \right) 1_{\rho>c^*}$$
Case $\alpha = \beta$: Leverage and Greed

Assume that $x_0 < E(\rho B)$ and that $c^* > 0$ is one of the minimizers in (1)

- $X^*_l = \left( \frac{x^* - (x_0 - E[\rho B])}{E[\rho 1_{\rho > c^*}]} - B \right) 1_{\rho > c^*}$
- Compute

$$\frac{x^* - (x_0 - E[\rho B])}{E[\rho 1_{\rho > c^*}]} - B = \left( \frac{a E[\rho B]}{E[\rho 1_{\rho > c^*}]} - B \right) - \frac{a x_0}{E[\rho 1_{\rho > c^*}]}$$

where $a := \frac{k(c^*)^{1/(1-\alpha)}}{k(c^*)^{1/(1-\alpha)}-1} > 1$
Case $\alpha = \beta$: Leverage and Greed

Assume that $x_0 < E(\rho B)$ and that $c^* > 0$ is one of the minimizers in (1)

- $X^*_l = \left( \frac{x^* - (x_0 - E[\rho B])}{E[\rho 1_{\rho>c^*}]} - B \right) 1_{\rho>c^*}$
- Compute

$$
\frac{x^* - (x_0 - E[\rho B])}{E[\rho 1_{\rho>c^*}]} - B = \left( \frac{a E[\rho B]}{E[\rho 1_{\rho>c^*}]} - B \right) - \frac{a x_0}{E[\rho 1_{\rho>c^*}]}
$$

where $a := \frac{k(c^*)^{1/(1-\alpha)}}{k(c^*)^{1/(1-\alpha)} - 1} > 1$

- Then the leverage

$$
L = \frac{E(\rho X^*_l)}{x_0} = \frac{1}{x_0} E \left[ \rho \left( \frac{x^* - (x_0 - E[\rho B])}{E[\rho 1_{\rho>c^*}]} - B \right) 1_{\rho>c^*} \right]
$$

$$
\geq (a - 1) \frac{E(\rho B)}{x_0} - a
$$

$$
= (a - 1)G - a \to +\infty \text{ as } G \to +\infty.
$$
Case $\alpha = \beta$: Potential Losses and Greed

The potential loss

\[
l = E\left(\frac{\rho X^*_l}{x_0} \mid X^* < B\right) = E\left(\frac{\rho X^*_l}{x_0} \mid \rho > c^*\right)
\]

\[
= \frac{E(\frac{\rho X^*_l}{x_0})}{P(\rho > c^*)}
\]

\[
\geq \frac{(a - 1)G - a}{P(\rho > c^*)} \rightarrow +\infty \text{ as } G \rightarrow +\infty.
\]
Case $\alpha = \beta$: Results

**Theorem.** (Jin and Zhou 2009) Assume that $x_0 < E(\rho B)$ and that $c^* > 0$ is one of the minimizers in (1). Then we have the following conclusions:

(i) $L \rightarrow +\infty$ as $G \rightarrow +\infty$.

(ii) $P(X^* < B) \equiv P(\rho > c^*)$ is independent of $G$.

(iii) $l \rightarrow +\infty$ as $G \rightarrow +\infty$. 
Case $\alpha < \beta$

- $\alpha < \beta$: loss aversion in a different (bigger) scale
Case $\alpha < \beta$

- $\alpha < \beta$: loss aversion in a different (bigger) scale
- Abdellaoui (2000): median of $\alpha$ and $\beta$ are 0.89 and 0.92 respectively
Case $\alpha < \beta$

- $\alpha < \beta$: loss aversion in a different (bigger) scale
- Abdellaoui (2000): median of $\alpha$ and $\beta$ are 0.89 and 0.92 respectively
- No solution provided in Jin and Zhou (2008) for this case
Case $\alpha < \beta$

- $\alpha < \beta$: loss aversion in a different (bigger) scale
- Abdellaoui (2000): median of $\alpha$ and $\beta$ are 0.89 and 0.92 respectively
- No solution provided in Jin and Zhou (2008) for this case
- Probability of loss occurrence now depends on $B$ or $G$
Case $\alpha < \beta$: A Critical Point

Denote

$$h(c) := \frac{k-T_-(1-F(c))}{(E[\rho 1_{\rho>c}]^\beta}, \quad c > 0$$

$$c_1 := \sup\{c' \in [0, +\infty) : h(c') = \inf_{c \in [0, +\infty)} h(c)\}$$
Case $\alpha < \beta$: A Critical Point

Denote

$$h(c) := \frac{k-T-(1-F(c))}{(E[p1_{p>c})]^\beta}, \quad c > 0$$

$$c_1 := \sup\{c' \in [0, +\infty) : h(c') = \inf_{c \in [0, +\infty)} h(c)\}$$

Some proved facts:
Case $\alpha < \beta$: A Critical Point

Denote

$$h(c) := \frac{k-T_-(1-F(c))}{(E[\rho1_{\rho>c}]^\beta}, \ c > 0$$

$$c_1 := \sup\{c' \in [0, +\infty) : h(c') = \inf_{c \in [0, +\infty)} h(c)\}$$

Some proved facts:

- Behavioural portfolio problem is well-posed if and only if
  $$\liminf_{c \to +\infty} h(c) > 0$$
Case $\alpha < \beta$: A Critical Point

Denote

$$h(c) := \frac{k - T(1 - F(c))}{(E[\rho 1_{\rho > c}])^\beta}, \quad c > 0$$

$$c_1 := \sup\{c' \in [0, +\infty) : h(c') = \inf_{c \in [0, +\infty)} h(c)\}$$

Some proved facts:

- Behavioural portfolio problem is well-posed if and only if
  \[ \liminf_{c \to +\infty} h(c) > 0 \]
- \( c_1 > 0 \) if \( \liminf_{c \to +\infty} h(c) > 0 \)
Case $\alpha < \beta$: A Critical Point

Denote

$$h(c) := \frac{k - T_-(1 - F(c))}{(E[\rho 1_{\rho > c}])^\beta}, \quad c > 0$$

$$c_1 := \sup\{c' \in [0, +\infty) : h(c') = \inf_{c \in [0, +\infty)} h(c)\}$$

Some proved facts:

- Behavioural portfolio problem is well-posed if and only if
  $$\liminf_{c \to +\infty} h(c) > 0$$

- $c_1 > 0$ if $\liminf_{c \to +\infty} h(c) > 0$

- Portfolio problem admits no optimal solution if $c_1 = +\infty$
Case $\alpha < \beta$: Results

**Theorem.** (Jin and Zhou 2009) Assume that $x_0 < E(\rho B)$, \(\liminf_{c \to +\infty} h(c) > 0\), and $0 < c_1 < +\infty$. Then portfolio problem admits optimal solution with a sufficiently large agent greed $G$. Furthermore, if $(c(G), x_+(G))$ is an optimal solution for the mathematical programme, then optimal terminal wealth is

$$X^* = \frac{x_+(G)}{\varphi(c(G))} \left( \frac{T_+(F(\rho))}{\rho} \right)^{1/(1-\alpha)} - \frac{x_+(G) - (x_0 - E[\rho B])}{E[\rho 1_{\rho > c(G)}]} 1_{\rho \leq c(G)} \frac{1_{\rho > c(G)} + B}{1_{\rho > c(G)} + B}.$$

Moreover,

$$\lim_{G \to +\infty} c(G) = c_1, \quad \lim_{G \to +\infty} x_+(G) = +\infty, \quad \lim_{G \to +\infty} \frac{x_+(G)}{G} = 0.$$

Finally, $L \to +\infty$ as $G \to +\infty$ and $l \to +\infty$ as $G \to +\infty$. 

Xunyu Zhou

Greed, Leverage, and Potential Losses: A Prospect Theory Perspective
Chance vs Scale of Losses

- Asymptotic probability of having gains is $P(\rho \leq c_1)$, independent of $G$
Asymptotic probability of having gains is $P(\rho \leq c_1)$, independent of $G$.

Agent gambles on an event with *positive* probability of occurrence (since $0 < c_1 < +\infty$).
Asymptotic probability of having gains is $P(\rho \leq c_1)$, independent of $G$.

Agent gambles on an event with positive probability of occurrence (since $0 < c_1 < +\infty$).

Asymptotic probability of having losses is also independent of $G$; however, scale of losses is catastrophic when greed is sufficiently strong.
A Model with Loss Control

- We have established both leverage and potential losses grow to infinity as greed expands to infinity.
A Model with Loss Control

- We have established both leverage and potential losses grow to infinity as greed expands to infinity.
- ... which suggests, from loss-control or regulatory perspective, a model with *a priori* bound on potential losses.
A Model with Loss Control

- We have established both leverage and potential losses grow to infinity as greed expands to infinity.
- ... which suggests, from loss-control or regulatory perspective, a model with a priori bound on potential losses.
- It would (indirectly) limit leverage and hence magnitude of greed.
A Model with Loss Control

- We have established both leverage and potential losses grow to infinity as greed expands to infinity
- ... which suggests, from loss-control or regulatory perspective, a model with *a priori* bound on potential losses
- It would (indirectly) limit leverage and hence magnitude of greed
- The new model (Jin, Zhang, Zhou 2009)

Maximize \( V(X - B) \)
subject to \[
\begin{align*}
    E[\rho X] &= x_0 \\
    X &\geq B - L \\
    X &\text{ is an } \mathcal{F}_T - \text{random variable}
\end{align*}
\]
Epilogue: Human Flaws Must be Contained

■ The 2008 financial crisis is testament to human flaws and limitations (greed, fear, euphoria, panic, skulduggery ... and, always-blame-others)
Epilogue: Human Flaws Must be Contained

- The 2008 financial crisis is testament to human flaws and limitations (greed, fear, euphoria, panic, skulduggery ... and, always-blame-others)
- Market could be spectacularly wrong, and hits everyone of us consequently
The 2008 financial crisis is testament to human flaws and limitations (greed, fear, euphoria, panic, skulduggery ... and, always-blame-others)

Market could be spectacularly wrong, and hits everyone of us consequently

Nothing wrong with financial conventions and innovations (mark-to-market, MBS, CDO, CDS, etc.); nothing wrong with human flaws;
Epilogue: Human Flaws Must be Contained

- The 2008 financial crisis is testament to human flaws and limitations (greed, fear, euphoria, panic, skulduggery ... and, always-blame-others)

- Market could be spectacularly wrong, and hits everyone of us consequently

- Nothing wrong with financial conventions and innovations (mark-to-market, MBS, CDO, CDS, etc.); nothing wrong with human flaws; what's wrong is human flaws *uncontrolled*
Epilogue: Human Flaws Must be Contained

- The 2008 financial crisis is testament to human flaws and limitations (greed, fear, euphoria, panic, skulduggery ... and, always-blame-others)
- Market could be spectacularly wrong, and hits everyone of us consequently
- Nothing wrong with financial conventions and innovations (mark-to-market, MBS, CDO, CDS, etc.); nothing wrong with human flaws; what's wrong is human flaws uncontrolled
- Regulations and interventions necessary - although a subtle balance important
Epilogue: Human Flaws Must be Contained

- The 2008 financial crisis is testament to human flaws and limitations (greed, fear, euphoria, panic, skulduggery ... and, always-blame-others)
- Market could be spectacularly wrong, and hits everyone of us consequently
- Nothing wrong with financial conventions and innovations (mark-to-market, MBS, CDO, CDS, etc.); nothing wrong with human flaws; what’s wrong is human flaws *uncontrolled*
- Regulations and interventions necessary - although a subtle balance important
- Behavioural finance a promising area in helping with re-building sound post-crisis financial infrastructure