Asset Allocation with Gross Exposure Constraints for Vast Portfolios with High Frequency Data

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Markowitz portfolio allocation problem:

\[
\begin{align*}
\text{min} & \quad w^T \Sigma w \\
\text{s.t.} & \quad w^T \mu \geq \mu_b \\
& \quad w^T 1 = 1
\end{align*}
\]  

(1)

where \( \Sigma = \text{var}(R) \).
Motivation

It is a simple quadratic programming problem with linear constraint. However, the solution produced by the typical low frequency approach has many problems. For example, it tends to produce extreme long and short positions which makes the portfolio unstable.
Motivation

Fan, Zhang and Yu (2008) showed that, using the daily closing price data, the desired portfolio features can be achieved by adding the $L - 1$ norm constraint to the original problem.

$$\min \ w^T \Sigma w$$
$$s.t. \quad w^T 1 = 1$$
$$\|w\|_1 \leq c$$
Motivation

We would like to explore the use of high frequency data to further improve the portfolio allocation.

In the previous literature, high frequency data has only been studied on a financial econometrics level, but never on a financial engineering level, which means that it is rarely used to make portfolio allocation decisions.
Motivation

Our goal is to see if, by using high frequency data, we can shorten the scale of the time window we need to estimate the covariation structure and improve the asset allocation.
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For asset price processes $S_t$, the log prices $X_t = \ln S_t$ follow the diffusion processes $dX_t = \mu_t dt + \sigma_t dB_t$.

We would like to minimize

$$\text{var}_t\left(\int_t^{T+t} w^T dX_u\right) = \mathbb{E}_t\left(\int_t^{T+t} w^T \sigma_u \sigma'_u w du\right)$$

$$= w^T \mathbb{E}_t\left(\int_t^{T+t} \Sigma_u du\right) w \quad (3)$$

where $\Sigma_u = \sigma_u \sigma'_u$. 
The realized volatility $\int_{t-h}^{t} \sigma_u \sigma_u' du$ is used to approximate the conditional expectation of the future realized volatility $E_t(\int_{t}^{T+t} \Sigma u du)$.

We are facing two major challenges, non-synchronous trading and microstructure noise in high frequency data.
Problem Setting

In reality, microstructure noise cannot be neglected. The log asset price processes $X_t$ are actually driven by underlying processes $Y_t$, which follow the diffusion processes $dY_t = \mu_t dt + \sigma_t dB_t$.

Zhang (2006) suggested the TSRC (Two Time-Scale Realized Covariation) approach to deal with the issue. Barndorff-Nielsen, Hansen, Lunde and Shephard (2008) and others suggested alternative approaches. We applied the former one. To fixe ideas, TSRC can be viewd as a modified version of the realized covariance of $Y$, which is $\sum_{j=1}^{n}[Y_{t_j} - Y_{t_{j-1}}][Y_{t_j} - Y_{t_{j-1}}]'$. 
Problem Setting

To deal with non-synchronous trading, we use the concept of "refresh time" introduced by Barndorff-Nielsen, Hansen, Lunde and Shephard (2008).
In terms of volatility estimation, we proposed the pairwise-refresh-time estimator, and compared it with the all-refresh-time estimator.

For all-refresh-time estimator, as the number of assets increases, the frequency of the refresh times is decreasing and a large amount of data is likely to be thrown away. It will not be a problem for pairwise-refresh-time estimator, which improves the precision of estimation.
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Let us briefly revisit the portfolio optimization problem with the $L - 1$ norm constraint:

\[
\begin{align*}
\min & \quad w^T \Sigma w \\
\text{s.t.} & \quad w^T 1 = 1 \\
& \quad \|w\|_1 \leq c
\end{align*}
\]
Risk Characteristics and Asymptotics

Let

\[ R(w) = w^T \Sigma w, \quad R_n(w) = w^T \hat{\Sigma} w \]

be respectively the theoretical and empirical portfolio risks.

And let

\[ w_{opt} = \arg\min_{w^T 1 = 1, \|w\|_1 \leq c} R(w), \quad \hat{w}_{opt} = \arg\min_{w^T 1 = 1, \|w\|_1 \leq c} R_n(w) \]

be respectively the theoretical optimal allocation vector we want and empirical optimal allocation vector we get.
We are interested in the behaviors and asymptotics of
\[ |R(\hat{w}_{opt}) - R(w_{opt})|, \quad |R(\hat{w}_{opt}) - R_n(\hat{w}_{opt})| \quad \text{and} \]
\[ |R(w_{opt}) - R_n(\hat{w}_{opt})|. \]

The theorems are under derivation.
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**Figure:** Comparison between all-refresh and pairwise-refresh approaches
Empirical Studies

Figure: Comparison between the high frequency approaches and low frequency approach
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Figure: Comparison between all-refresh and pairwise-refresh approaches

actual annualized volatility (%) vs exposure parameter (c)
Simulation

Figure: Comparison between the high frequency approaches and low frequency approach