Auction Theory
(what we’d expect in a fully-rational, well-behaved world)

Single item, varying tastes (and risk-neutral bidders)

The Revenue Equivalence Theorem. Classical result in the theory of auctions about the division of expected social surplus among risk-neutral (when marginal utility of income does not change when income increases) bidders and a risk-neutral bid-taker. Whenever the bidders have independent private valuations for the resource in sale, all auction formats lead to the same expected revenue to the bid-taker, and to the same expected profits of the bidders, which award the object to the bidder that submits the highest bid - regardless of the specific payment rule of the auction. In particular, the equilibrium expected payments in the first price sealed bid auction or the Dutch auction are the same as in the second price sealed bid auction, in the English auction, or in any all pay auction. The revenue equivalence theorem shows that in terms of the objective functions of risk neutral strategic traders which have independent private information, all 'reasonable' auction formats are equivalent exchange mechanisms. This equivalence extends to auctions of multiple identical goods if the bidder have unit demands. It does not hold, however, in common value auctions, with risk-averse traders, or in auction markets of multiple goods when the bidders bid for more than one item.

(from Jan Vleugels)

Paul Klemperer gives the following more formal statement (and a complete treatment in Appendix A) in his paper "Auction Theory: A Guide to the Literature". (Journal of Economic Surveys v13, n3 (July 1999): 227-86):

Assume each of a given number of risk-neutral potential buyers of an object has a
privately-known signal independently drawn from a common strictly-increasing, atomless distribution. Then any auction mechanism in which

1. the object always goes to the buyer with the highest signal, and

2. any bidder with the lowest-feasible signal expects zero surplus

yields the same expected revenue (and results in each bidder making the same expected payment as a function of her signal).

The theorem applies very broadly to auction types, beyond the English, Dutch, First- and Second-Price auctions, to include nearly any "reasonable" auction mechanism, and even some peculiar mechanisms such as the All-Pay auction.

Klemperer also points out that the theorem can apply in common value auctions when the bidders' signals are independent, and that, "The theorem extends to the case of $k > 1$ indivisible objects being sold, provided bidders want no more than one object each; all auctions that give the objects to the $k$ highest-value bidders are revenue equivalent."

... 

- All yield (at the symmetric equilibrium) the same expected payoff to the seller.

1. The optimality of threats and extortion (i.e., reservation prices and entry fees)

- By threatening an inefficient outcome, the seller gains.

**Single item, some unknown attributes**
2. The Linkage Principle (“Who’s Afraid of the Winner’s Curse?”)

- The Winner’s Curse hurts the seller(!)
  … so link the final price to the information held by others

- English > Second-price > Dutch = First-price

3. Honesty is the best policy

- Repeat sellers do best by fully and accurately revealing all they know.

Glossary of Auction Terms

This glossary contains auction terms that are commonly used electronic and traditional format auctions. These terms are provided as an aid to exploring the world of Internet auctions and auction theory.

**Auction**- An auction is a formal method for allocating goods or services based on competition. An auction is driven by two basic principles: 1) the seller wanting to obtain the maximum price, and 2) the buyer wanting to pay the minimum price.

**Bidding increment**- The bidding increment refers to the amount of change that is to occur to the standing bid. A minimum bidding increment is often set by the auctioneer to establish a good pace for the auction. In some auctions, the bidder establishes an increment that exceeds the auctioneer's bidding increment to hasten the closure of the auction, to signal a high-level of interest, or simply to intimidate other participants.

**Bid Max**- This is a feature allowing a bidder to enter an amount that they are willing to pay, and sets a maximum bid that is not to be exceeded. This allows the bidder to have their bid automatically raised to the lowest possible winning bid. If someone
places a bid higher than your bid max you will lose the auction but you will be in the auction up until that point. This is solely a matter of convenience for the bidders who choose to use it.

**Closed auction**- This term is sometimes used to indicate that bids are no longer being accepted because the auction has already concluded. At other times, the term is used to refer to a private-bid auction.

**Minimum price**- This term is sometimes used interchangeably with reserve price to indicate a minimum price that the seller will accept for the item being auctioned. The distinction between reserve and minimum is that minimum price sellers are committed to sell at the minimum price, but have the option to go below that price at the conclusion of the sale to honor the bid of the highest bidder. Reserve price sellers end the auction without a sale if the reserve price is not met.

**Multiple-item auction**- A term used to indicate that multiple quantities of an item are being placed into auction. Dutch auctions are similar, since they involve multiple quantities of the same item. In some multiple-item auctions, the price specified in the individual bid is used to determine what winners pay. (See discriminatory auction.) In other multiple-item auctions, all winners pay me price specified in the lowest winning bid. In some auctions, quantities being bid upon have an effect. Those ordering larger quantities may be declared winners, or there may even be a different reserve price for different quantities.

**Online auction**- This term refers to auctions that are conducted by electronic means, rather than at a physical location (at an estate or auction house). The Internet offers an increasingly popular venue for conducting electronic auctions.

**Private auction**- This auction is only opened to select bidders. Private auctions may be "invite only" or require some special qualifications in order to bid. (See closed-bid auction.)

**Public auction**- During a public auction anyone is allowed to place a bid. The public auction technique adds to the excitement and maximizes the competitive nature of bidders.

**Reverse-bid auction**- This type offer auction uses the bid-down of principle. The reverse bid auction may also have a reserve price in effect, meaning that the buyer will not accept any bid that exceeds that price.

**Single-item auction**- In this type of auction, a single item, rather than multiple units of the same item, is being offered for sale.

**Straight auction** - This is a term commonly used to describe an auction that uses the bid-up principle. In the straight auction, the winner is the highest bidder, and pays the price that he bid. The straight auction may have a minimum bid or reserve price in effect.

### Auction Theory

**Index:**

- Introduction
• First price auctions with independent private values
• Second price auctions with independent private values
• The revenue equivalence theorem
• Common values and the winner’s curse.

Introduction

• Well specified rules
• Easily identifiable players with visible objectives

   Hence, auctions are a class of mechanisms

Auction rules concern:

• Who is allowed to play (open, closed)
• What is an acceptable bid (reserve price)
• What is the method of bidding:
  Sealed bid, ascending or descending
• What do you bid for
• Who wins the auction and what price he must pay

First price and second price auctions

Basic Model
1. There is a single unit to be sold
2. There are two buyers.
A buyer’s valuation for the unit is $2H$, and the surplus he has if he gets the object is $2P$, where $P$ is the price he pays.
Buyers can be of two equally probable types, $\{2H, 2L\}$, with $2H > 2L$
   (Terminology=Independent private values case)

The seller doesn’t know the valuation of buyers.
Each buyer doesn’t know the valuations of the other buyers.

SECOND PRICE AUCTIONS.
In a second price auction, the winner is the player with the highest bid and the price he pays is the second highest bid.
Result 1: In a second price auction bidding on one’s valuation is a dominant strategy.
Hence:
\[ 2_H \text{ bids } B=2_H \]
and \[ 2_L \text{ bids } B=2_L \]
The reason for this is that what a player pays does not depend on what he/she bids.
And the surplus of a \(2_H\) type is
\[ \frac{1}{2} (2_H - 2_L) + \frac{1}{4} (0) = \frac{1}{2} (2_H - 2_L) \]
while the surplus of a \(2_L\) type is 0.

**First Price Auction**

In a first price auction, the winner is the player with the highest bid and the price he pays is his bid.

1. **No asymmetric information** *

   The winner of the auction is a player with highest valuation.

   He either bids:

   \[ B=2_L + \] (if he is confronted with a low type) specifying the most general strategy - this is a mixed strategy which consists of:

   Bidding \( B \) with probability \( p(B) \).

   (Note that the variable of choice of the \( 2_H \) type is the probability function \( p(B) \).)

   Note that this is quite general.

   For example:

   If the bidder wants to bid \( B_0 \) then \( p(B_0) = 1 \) and \( p(B) = 0 \), if \( B \) is not \( B_0 \).

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**Finding \( p(B) \).**

The bidder wants to select the optimal \( p(B) \). This is to choose \( p(B) \) that maximizes his expected revenue.

**Method 1**

Choosing \( p(B) \) is like choosing a mixed strategy.

Remember: In any game if a player decides to randomize between pure strategies this implies that he is indifferent between these pure strategies.

Here it is the same: a player randomizes between \( B \) and \( B' \) whenever they yield the same payoff.

What is the player’s payoff if \( B=2_L \)?

It is \( \frac{1}{2} (2_H - 2_L) \)

Hence, whenever the player randomizes between \( B=2_L \) and \( B'>2_L \) the payoff of both options will be: \( \frac{1}{2} (2_H - 2_L) \).

What is the player’s payoff if \( B=B' \)?
½ (2_H - B') + ½ prob(b<B')*(2_H - B') =
½ (2_H - B') [1+ prob(b<B')]

Hence,
½ (2_H - B') [1+ prob(b<B')] = ½ (2_H - 2_L)

which implies that:
prob (b<B') = (B' - 2_L)/(2_H - B')

Implications:

- 0 probability to B=2_H
- maximum B=(2_H+2_L)/2
- The player plays higher B’s with higher probabilities.

MOREOVER: what is the surplus of the 2_H type?
And of the 2_L type?

THE EQUIVALENCE REVENUE THEOREM

In an independent private values environment with risk neutral bidders, the expected price paid for the object is the same under a first price or a second price sealed bid auction.

1. risk averse bidders
2. common value

Common value and the winner’s curse
All bidders true value of the good is equal. However, players are generally uninformed about this true value.
In a common value auction, the bidder that wins the auction is generally the one that overestimates the value of the object.
In making bids, players should take this into account. However, empirical research shows that generally bidders that win make a loss. The failure to update beliefs of the winner is known as the winner's curse

Good slide show on Auction
http://www.wisdom.weizmann.ac.il/home/bennyp/public_html/presentation

Rules representing the end of auctions:

An important issue in auction design concerns the rules governing the end of the auction. The internet auctions conducted by eBay and Amazon present a natural experiment because they use different rules for ending an auction.

Auctions on eBay have a fixed end time, while auctions on Amazon, which operate under otherwise similar rules, do not have a fixed end time, but
continue if necessary past the scheduled end time until ten minutes have passed without a bid.

The strategic differences in the auction rules are reflected in the auction data by significantly more late bidding on eBay than on Amazon. Furthermore, more experienced bidders on eBay submit late bids more often than do less experienced bidders, while the effect of experience on Amazon goes in the opposite direction. On eBay, there is also more late bidding for antiques than for computers.

We also find scale independence in the distribution over time of bidders' last bids, of a form strikingly similar to the 'deadline effect' noted in bargaining: last bids are distributed according to a power law.

Both the theory and the data suggest that multiple causes contribute to late bidding, with strategic issues related to the rules about ending the auction playing an important role. Controlled experiments are needed (and are underway) to better understand the design implications of these results.

The basic rules of Amazon and eBay auctions

- **Second price** auctions (bidders may submit their reservation price (also called a proxy bid); the
resulting bid registers as the minimum increment above the previous high bid, and rises until it is reached, or until it exceeds the minimum increment over the next highest reservation price). The winning bidder is therefore the one who has submitted the highest reservation price, and the price he pays is the minimum increment over the second highest reservation price.

- Auctions typically run for several days.
- **eBay auctions have a fixed deadline.**
- **Amazon auctions are automatically extended for an additional 10 minutes from the time of the latest bid.**

**Multiple and late bidding in eBay auctions**

- Many bidders submit multiple bids in the course of the auction (i.e. they submit and later raise the reservation price they authorize for proxy bidding on their behalf).
- A non-negligible fraction of bids are submitted in the closing seconds of the auction (a practice called "sniping").
For example, in an exploratory sample of just over 1000 completed eBay auctions sampled in May and June 1999, 28% had 0 bidders, 16% had exactly 1 bidder, and of the remaining 585 auctions, 74% showed multiple bidding (at least one bidder raised his reservation price in the course of the auction), and 18% had bids in the last sixty seconds. There is substantial variation in the percentage of last-minute bids. The highest percentage in this sample was in the category Antiques—Ancient World, in which 56% of the auctions that attracted more than one bid had bids in the last sixty seconds, while the lowest such percentage was in Collectibles: Weird Stuff: Totally Bizarre, where it was 0%. 
IBM Thinkpad 600E PII-366/64MB/6G/24X/56

**Item #302534742**

<table>
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<tr>
<th>Description</th>
<th>Value</th>
<th>Additional Information</th>
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<tr>
<td>Currently</td>
<td>$1,556.00</td>
<td>First bid $</td>
</tr>
<tr>
<td>Quantity</td>
<td>1</td>
<td># of bids 2</td>
</tr>
<tr>
<td>Time left</td>
<td>3 hours, 8 mins +</td>
<td>Location</td>
</tr>
<tr>
<td>Started</td>
<td>Apr-06-00 16:20:59 PDT</td>
<td>Country</td>
</tr>
<tr>
<td>Ends</td>
<td>Apr-11-00 16:20:59 PDT</td>
<td></td>
</tr>
<tr>
<td>Seller (Rating)</td>
<td>vincelee (192)</td>
<td>(mail this auction)</td>
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<tr>
<td>Watch this item</td>
<td></td>
<td>(request a gift)</td>
</tr>
<tr>
<td>High bid</td>
<td>kb8ho (0)</td>
<td>(view comments in seller's Feedback Profile)</td>
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<td>(view seller's other auctions)</td>
</tr>
<tr>
<td>Shipping</td>
<td>Buyer pays fixed shipping charges, Seller ships internationally (worldwide)</td>
<td></td>
</tr>
</tbody>
</table>

Seller assumes all responsibility for listing this item. You should contact the seller to resolve any questions unless otherwise noted.
Bidding

IBM Thinkpad 600E PII-366/64MB/6G/24X/56K-A++
Item #302534742

Current bid: $1,556.00
Bid increment: $25.00

Your maximum bid: $2,582.00

(Minimum bid: $1,582.00)

eBay will bid incrementally on your behalf up to your maximum bid, which is kept secret from other eBay users. The eBay term for this is proxy bidding.

Your bid is a contract - Place a bid only if you're serious about buying the item. If you are the winning bidder, you will enter into a legally binding contract to purchase the item from the seller.

Bid history of completed auction
eBay Bid History for IBM Thinkpad 600E PII-366/64MB/6G/24X/56K~A++ (item #30253)

If you have questions about this item, please contact the seller at the User ID provided below. Seller assumes all responsibility for listing this item.

Last bid for this item: $1,626.00
Date auction ends: Apr-11-00 16:20:59 PDT
Date auction started: Apr-06-00 16:20:59 PDT
Seller: vincentlee (192)
First bid at: $8.01
Number of bids made: 23 (may include multiple bids by same bidder)

Bidding History (in order of bid amount):

cjennnings@aol.com (-1)
Last bid at: $1,626.00
Date of bid: Apr-11-00 16:20:59 PDT

skatcher (0)
Last bid at: $1,601.00
Date of bid: Apr-11-00 16:04:15 PDT

wbzhe (0)
Last bid at: $1,556.00
Date of bid: Apr-09-00 11:17:33 PDT

nextracse (0)
Last bid at: $1,550.01
Date of bid: Apr-10-00 11:37:19 PDT

SK210 (2)
Last bid at: $1,400.00
Pokemon — Holo Charizard! Holofoil!

Bidding is closed for this item.

Current bid: $26.50
Quantity: 1
Time left: Auction has ended

Seller (Rating): aer51 (25)

High Bid: jacke5 (11)
Payment: Money Order/Cashier Checks, Personal Checks, See item description for payment methods accepted
Escrow: Accepted. Buyer will pay for escrow

eBay Online Payments by Billpoint

Instant Purchase Available

*High Bidder: Click here to view Order Details.
*Seller: Click here to view/update Order Details.
16 bids by 7 bidders, last bid about 45 seconds before end of auction
9 bids by 6 bidders, last bid (but not highest bid) about 3 seconds before end of auction.

The dangers of last-minute bidding

eBay.com's view (1999)

"Bid Sniping (last minute bidding)

eBay always recommends bidding the absolute maximum that one is willing to pay for an item early in the auction. eBay uses a proxy bidding system. (...) Thus, if one is outbid, one should be at worst, ambivalent toward being outbid. After all, someone else was simply willing to pay more than you wanted to pay for it. If someone does outbid you toward the last minutes of an auction, it may feel unfair, but if you had bid your maximum amount up front
and let the Proxy Bidding system work for you, the outcome would not be based on time."

*  

A seller’s view (Axis Mundi, 1999)
"THE DANGERS OF LAST MINUTE BIDDING: Almost without fail after an auction has closed we receive emails from bidders who claim they were attempting to place a bid and were unable to get into eBay. There is nothing we can do to help bidders who were "locked out" while trying to place a "last minute" bid. All we can do in this regard is to urge you to place your bids early. If you're serious in your intent to become a winning bidder please avoid eBay's high traffic during the close of an auction. Its certainly your choice how you handle your bidding, but we’d rather see you a winner instead of being left out during the last minute scramble."

*  

AuctionWatch.com’s view (1999)
"There are inherent risks in sniping. If you wait too long to bid, the auction could close before your bid is processed. If your maximum doesn't beat the current high bidder, you won't have a second chance to up the ante. And don't overlook the fact that you could be in the company of other snipers who are ready to snipe your snipe. It happens all the time."

What does the existing theory say?

• **Independent private value auctions**: It is a dominant strategy for bidders in a second-price sealed-bid auction to submit their maximum willingness-to-pay.

• **Common value auctions**: Bidders can get information about others’ bids that causes them to revise their willingness to pay (see Bajari and Hortacsu, 2000).

• **Non-strategic behavior**: naive time dependence, learning, endowment effect, …
The economics literature that has started to be directed at these phenomena favors the latter two explanations—irrationality, or common value auctions. Several papers assert that it is a dominant strategy for eBay bidders to submit their true willingness to pay in a private value auction, and at least one paper takes the prevalence of late bidding to be strong evidence that the auctions in which it occurs are common value auctions.

The theoretical contribution of this paper will be to show that, under eBay rules, late bidding can occur, at equilibrium, in private value as well as common value auctions, in a way that it cannot under Amazon rules. Also, late bidding is a rational strategic response to certain kinds of naïve behavior under eBay rules, but not under Amazon rules. This will guide the empirical investigation into whether the observed late bidding is consistent with sophisticated strategic behavior (e.g. adaptive behavior of experienced bidders).

A strategic model of the eBay environment (without bidder valuations yet)

- \( n \) bidders, \( N = \{1, \ldots, n\} \)
- Minimum initial bid \( m \) and smallest increment \( s \).
The "current price" (or "high bid") in an auction with at least two bidders is in general equal to the minimum increment over the second highest submitted bidders’ reservation price. There are two exceptions to this rule.

- If more than one bidder submitted the highest reservation price, the bidder who submitted her bid first is the high bidder at a price equal to the reservation price.
- Second, if the current high bidder submits a new, higher bid, the current price is not raised, although the number of bids is incremented.

Each submitted reservation price must exceed both the current high bid and the bidder’s last submitted reservation price.

A player can bid at any time $t \in [0, 1)$. A player has time to react before the end of the auction to another player’s bid at time $t' < 1$, but the reaction cannot be instantaneous, it must be strictly after time $t'$, at an earliest time $t_n$, such that $t' < t_n < 1$. At $t = 1$, everyone knows the bid history prior to $t$, and has time to make exactly one more bid, without knowing what other last minute bids are being placed simultaneously.

- If two bids are submitted simultaneously at the same instant $t$, then they are randomly ordered, and each has equal probability of being received first.
- At time $t = 1$, the probability that a bid is successfully transmitted is $p < 1$ (before that, $p=1$).

**Common value auctions**

Late bids in symmetric common value auctions arise either so that bidders can incorporate into their bids the information they have gathered from the earlier bids of others, or so bidders can avoid giving information to others through their own early bids.

The following example illustrates this kind of equilibrium late bidding. It is motivated by auctions of antiques, in which there may be bidders who are dealer/experts who are better able to identify high value antiques,
but who do not themselves have the highest willingness to pay for these once they are identified.

A "dealer/expert" model of antique auctions;

The object for sale has one of two conditions, "Fake," with probability \( p_F \), or "Genuine," with probability \( p_G = 1 - p_F \). There are two (representative) bidders. The first, U, is uninformed but discerning, and values Genuines more highly than Fakes (\( v_U(F) = 0 < v_U(G) = H \)), but can’t distinguish them, i.e. can’t tell whether the state of the world is F or G. The second, I, is informed (e.g. an expert dealer); with perfect knowledge of the state of the world, and values \( v_I(F) = 0 \) and \( v_I(G) = H - c \), with \( 0 < m < m + s < H - c < H \).

The common value aspect of this auction arises from the concern of the bidders with whether the object is Genuine. The strategic problem facing the informed bidder is that if the object is Genuine and he reveals this by bidding at any time \( t < 1 \), then the uninformed bidder has an incentive to outbid him. But the strategic problem facing the uninformed buyer is that, if he bids \( H \) without knowing if the object is Genuine, he may find himself losing money by paying \( m \) for a Fake.

**Theorem:** Late bidding in an eBay common value auction.
When the probability that the object is Fake is sufficiently high, at any sequentially rational equilibrium in undominated strategies in this "dealer/expert model" the uninformed bidder U does not bid, and the informed bidder bids only if the object is Genuine, in which case he bids \( v_l(G) = H-c \) at \( t=1 \). If the informed player deviates and makes a positive bid at any \( t<1 \), then the uninformed bidder bids \( H \) at some \( t' \) such that \( t<t'<1 \).

Private Value Auctions:

- True willingness to pay \( v_j \), distributed according to some bounded distribution \( F \).
- A bidder who wins the auction at price \( p \) earns \( v_j - p \).
- A bidder who does not win the auction earns 0.
- ["willingness to bid"] A bidder who earns 0 prefers to do it by bidding \( v_j \), and earning \( v_j - v_j = 0 \), rather than by losing the auction. (we will only use this in one of the Amazon results, but mention it early to show that this assumption we’ll add later doesn’t alter any of the earlier results).
Theorem: Late and multiple bidding in eBay private value auctions.
It is not a dominant strategy for each bidder $j$ to bid his true value $v_j$ at some time $t < 1$. There can exist equilibria in which bidders make multiple bids, but no bidder bids his true value until the last moment, $t = 1$, when there is only probability $p < 1$ that the bid will be transmitted.

Intuition.
There is a cost to relying on a last minute bid, since it has a positive probability that it will fail to register in the auction.

- But there will also be an incentive not to bid too high when there is still time for other bidders to react, to avoid a bidding war that will raise the expected final transaction price.

- The incentive for mutual delay until the last minute in eBay comes from the positive probability that another bidder’s last minute bid will not be successfully transmitted. Thus at this equilibrium, expected bidder profits will be higher (and seller revenue lower) than at the equilibrium at which everyone bids true values early.

- So at equilibrium, buyers may bid both early and at the very last moment.

(But there are lots of equilibria, including "bid vi at $t=0" )
Amazon has an automatic extension, rather than a "hard close."

Its rules are essentially like eBay's, except for the rule about how an auction ends. An Amazon auction ends either at the scheduled time if no bids have been placed in the last ten minutes, and otherwise it is extended until ten minutes have passed without a bid being received.

Our model of Amazon will be like our eBay model (min bid m, smallest increment s, etc.), except for the ending rule.

- The times $t$ at which a bid can be made are
  \[ [0, 1) \cup (1) \cup (1, 2) \cup (2) \cup (2, \infty) \cup (n-1, n) \cup (n, \infty) \ldots \]

- The auction extends past any time $\{n\}$ only if at least one bid is successfully transmitted at time $t = n$, and ends at the first $n$ at which no bid is successfully transmitted.

- Bids submitted before time $t = 1$, and in any open interval $(n - 1, n)$, are successfully transmitted with certainty.

- Bids submitted at time $t = 1$, and at any $t = n$ for $n = 1, 2, \ldots$, are successfully transmitted with probability $p < 1$.

**Theorem:** At a sequentially rational equilibrium in undominated strategies of an Amazon private value auction, the auction is not extended. All bidders bid their values before $t=1$.

**Sketch of proof:** At a sequentially rational equilibrium in undominated strategies:

1. **No bidder ever bids above his value:** Any strategy that calls for bidder $j$ to bid above $v_j$ at any time $t$ is dominated by the otherwise identical strategy in which $j$ bids at most $v_j$. 
2. The auction must end by stage \((n^*-1, n^*) \cup \{n^*\}\), with \(n^*\) defined by \(sn^* \leq v_{\text{max}} < s(n^* + 1)\), since the highest possible value will be exceeded after that, even if the current price never increases more than the minimum increment \(s\) at any round. (Here is where we use the boundedness of the distribution of values). If the auction gets to this last possible stage, 1 implies there is only room for the price to rise by \(< 2\) more increments \(s\).

3. If the auction gets to stage \((n^*-1, n^*) \cup \{n^*\}\), any remaining bidders who are not already the current high bidder and who have a value greater than the current price \(+ s\) will bid their true value at some \(t < n^*, \) i.e. at a time when \(p=1\). (Recall our willing-bidders assumption—here we use it to rule out possible indifference between bidding and not.)

4. Inductive step. Suppose at some stage \((n-1, n) \cup \{n\}\), it is known that at the next stage any remaining bidders who are not the current high bidder and who have a value greater than the current price \(+ s\) will bid their true value at a time when \(p=1\). Then (since a price war will result if the auction is extended by a successful bid at \(t=n\)) any strategy profile that calls for a bid at \(t=n\) is not part of an equilibrium, since that
bidder gets a higher expected return by bidding his value at \( t < n \), with \( p = 1 \).

5. The auction ends in the first stage: all bidders bid their true value by \( t < 1 \). (There is also no multiple bidding.) \{Strictly speaking we could conclude all bids are at \( t = 0 \), but the model would be better if it started with the open interval \((0,1)\)…\}

Common value auctions on Amazon:
The common value world is big, and can almost surely accommodate late bidding on Amazon. But some kinds of late bidding that occur at equilibrium on eBay won’t occur at Amazon equilibria.

Theorem: in the "expert/dealer model", at an equilibrium in undominated strategies, the dealer never wins an auction.

Sketch of proof: the dealer values Fakes at 0, and values Genuines less than the uninformed bidder. If the object is Fake, no dealer buys it, since a strategy at which the dealer bids a positive amount for a Fake is dominated by an otherwise identical strategy at which he doesn't. So if a dealer bids, he reveals the object is Genuine, and at equilibrium is outbid with \( p = 1 \) by the uninformed bidder.

Hypotheses about the causes of late bidding

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<thead>
<tr>
<th>Hypothesis</th>
<th>Predicted contribution</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>Strategic hypotheses</td>
<td>a. <em>Collusive equilibrium:</em> bidders bid late to avoid bidding wars with other like-minded bidders.</td>
</tr>
<tr>
<td>----------------------</td>
<td>-------------------------------------------------------------------------------------------------</td>
</tr>
<tr>
<td></td>
<td>• <em>Rational response to naïve English auction behavior:</em> bidders bid late to avoid bidding wars with naïve bidders.</td>
</tr>
<tr>
<td></td>
<td>• <em>Informed bidders protecting their information:</em> e.g. late bidding by &quot;expert/dealers.&quot;</td>
</tr>
<tr>
<td>Non-strategic hypotheses</td>
<td>Bidders bid late because they procrastinate, or because of naïve behavior, or because they don't like to leave bids &quot;hanging,&quot; or because search engines present soon-to-expire auctions first, etc.</td>
</tr>
</tbody>
</table>

### A Natural Experiment

A natural experiment arises because of differences in the auction rules of two large auction houses, eBay and Amazon. The experiment allows a preliminary test of naïve versus sophisticated bidding explanations of observed bidding behavior.

**Description of the data**
• Amazon and eBay publicly provide data about the bid history.
• We downloaded data in the categories "Computers" and "Antiques".
  o Computers (‘private value auctions’): information about the retail price of most items is in general easily available. The difference between the retail price and each bidder’s willingness to pay, however, is private information.
  o Antiques (‘common value auctions’): retail prices are usually not available and the value of an item is often ambiguous and sometimes require experts to appraise. As a consequence, the number and amounts of bids of others are likely to carry valuable information about the item’s value.

Our data set consists of randomly selected auctions that were completed between October 1999 and January 2000 and that met certain criteria.
  • At least two bidders
  • Auctions with a hidden reserve price only if the reserve price was met.
  • No Dutch-, Private-, "10% off 1\textsuperscript{st} bidder"-, or "Take-it-price"-auctions

In total, we randomly selected 480 auctions with 2279 bidders.
  • 120 eBay-Computers with 740 bidders
  • 120 Amazon-Computers with 595 bidders
  • 120 eBay-Antiques with 604 bidders
  • 120 Amazon-Antiques with 340 bidders

Variables
  • number of bids
  • number of bidders
  • reserve price (y/n)
  • "timing" of bidders’ last bids
• bidders’ "feedback numbers"

Timing

• For each bidder we downloaded the data about how many seconds before the deadline the last bid was submitted if the bid came in within the last 12 hours of the auction time.

• For each last bid in Amazon, we computed the number of seconds before a ‘hypothetical’ deadline.

• This hypothetical deadline is defined as the current actual deadline at the time of bidding under the assumption that the bid in hand and all subsequent bids were not submitted.

• Suppose, for example, a bid comes in two minutes before the initial deadline. Then, the auction is extended by 8 minutes. The bid shows up in our data, however, as 120 seconds, respectively.

Feedback number

• In eBay, buyers and sellers have the opportunity to give each other a positive feedback (+1), a neutral feedback (0), or a negative feedback (−1) along with a brief comment.

• The cumulative total of positive and negative feedbacks is what we call the "feedback number" in eBay.

• In Amazon, buyers and sellers are allowed to post 1-5 star ratings about one another.

• We refer to the cumulative number of ratings as the "feedback number" in Amazon.

Late bidding
Cumulative distribution of bidders’ last bids over time
When are the last bids submitted?

<table>
<thead>
<tr>
<th>bidder level</th>
<th>auction level</th>
</tr>
</thead>
<tbody>
<tr>
<td>eBay</td>
<td>eBay</td>
</tr>
<tr>
<td>Amazon</td>
<td>Amazon</td>
</tr>
<tr>
<td>Time Period</td>
<td>eBay 20%</td>
</tr>
<tr>
<td>-----------------</td>
<td>----------</td>
</tr>
<tr>
<td>Last hour</td>
<td>20 %</td>
</tr>
<tr>
<td>10 minutes</td>
<td>14 %</td>
</tr>
<tr>
<td>5 minutes</td>
<td>13 %</td>
</tr>
<tr>
<td>1 minute</td>
<td>8 %</td>
</tr>
<tr>
<td>10 seconds</td>
<td>2 %</td>
</tr>
</tbody>
</table>

Furthermore, more experience causes eBay bidders to bid later, but causes Amazon bidders to bid earlier.
A closer look at the timing of last bids in eBay

It is almost impossible to say whether a distribution of last bids is drawn from, say, the last 30 minutes or from the last 12 hours if no information about the time scale is given.

The figure shows empirical cdfs $F_X(t)$, where $F_X(t) = \Pr[X \leq t \leq T]$ is the probability that the last bid is submitted in the last $t$ seconds of the
auction, for six end intervals $[0, T]$ with $T = 12h, 6h, 3h, 1h, 30m, \text{and } 10m$.

*Scale independence*,

$$\frac{F_X(at)}{F_X(t)} = g(a) \quad \text{for all } a \in [0,1],$$

implies a cumulative distribution function of a power-functional form:

$$F_X(t) = \left(\frac{t}{T}\right)^\alpha.$$

**Empirical and estimated cdfs**

- dependent variable = the logarithm of $F(t)$ of the sample; explanatory variable = the logarithm of $t/T$ with $T = 12$ hours,

- OLS-estimations yield: $\alpha = 0.392$ for eBay Computers ($R^2 = 0.99$, Nobs = 301); $\alpha = 0.228$ for eBay Antiques ($R^2 = 0.99$, Nobs = 260).
Conclusions

Multiple and late bidding can have multiple causes.

- The clear difference in the amount of late bidding on eBay and Amazon is strong evidence that rational strategic considerations play a significant role, because eBay’s hard close gives more reason to bid late in private value auctions, in common value auctions, and against naïve incremental bidders. This evidence that rational considerations are at work is strengthened by the observation that the difference is even clearer among more experienced bidders.

- The difference between the amount of late bidding for computers and for antiques suggests that the bidders respond to the additional strategic incentives for late
bidding in markets in which expertise plays a role in appraising values.

- The substantial amount of late bidding observed on Amazon, (even though substantially less than on eBay) suggests that there are also non-strategic causes of late bidding, possibly due to naivete or other non-rational cause, particularly since the evidence suggests that it is reduced with experience.

- **The need for experiments**

  Many explanations may account for multiple and late bidding. These include not only equilibrium models as described above but also non-strategic behavior such as learning as suggested by parts of the field data described above. The field data, however, does not provide sufficient control to separate between the competing theories.

  - Experiments can control the market size and the degree of experience in different auction houses and therefore allow a cleaner comparative static analysis of bidding behavior under eBay and Amazon auction rules.

  - Experiments can eliminate common value aspects of the auctioned items.

  - Experiments allow observing individual multiple bidding.

  - Experiments can separate experience-based explanations from expertise-based explanations.

  - Experiments can reveal which auction house is more efficient and which yields higher seller revenues.

  - Experiments can control the effect of ‘noisy auction features’ like reserve prices, take-it-prices, 10% off-offers, offline transactions, offline communication and collusion.

**Experimental Environment:**
**Discrete eBay.**

**Stage 1:**
Discrete periods 1,…n,…
In each period, each trader has an opportunity to make a bid (simultaneously): the minimum acceptable bid is the smallest increment over the price at the previous period. At the end of each period, the high bidder and current price (minimum increment over second highest reservation price) are displayed to all. Stage 1 ends (only) after any period at which no player makes a bid. (Players can only make a finite number of potentially profitable bids, because of requirement that each new bid must exceed the previous high bid by the minimum increment. (If necessary, place ceiling on max bids, but should be higher than value, so as not to suppress excessive bids that Ss may wish to make…)

**Stage 2:** Simultaneous sealed (last) bid; probability p of being transmitted.

**Discrete Amazon:** Stage 1 followed by Stage 2, repeating after any successful Stage 2 bid.

**Sealed Bid:** stage 2 only (with p=1)

Experimental Design:

<p>| Ebay [p&lt;1 and p=1?] | Amazon | Sealed bid [ p=1] |</p>
<table>
<thead>
<tr>
<th>Private value</th>
<th>Late bids; lower revenue</th>
<th>Fewer late bids; higher revenue</th>
<th>Higher revenue</th>
</tr>
</thead>
<tbody>
<tr>
<td>Common value (with some informed bidders)</td>
<td>Late bids by informed bidders</td>
<td>Mixed strategy</td>
<td>? no opportunity to profit from information in the room—revenue depends on how info is distributed</td>
</tr>
<tr>
<td>Commodity (within subject, at end of other auctions)</td>
<td>Endowment effect?</td>
<td>Endowment effect?</td>
<td>Lower revenue than when endowment effect has time to work.</td>
</tr>
</tbody>
</table>

Analysis will also have to focus on experienced versus inexperienced bidders.

- Design questions raised by these results:

Does a fixed-deadline auction of a *private value* good raise less revenue than one with the same number of bidders in which bidding is extended until no one wants to bid anymore? (Because when many bidders plan to bid at the last minute, some don’t make it?)

Could the increased entertainment value of a fixed deadline attract sufficiently many bidders to overcome this?
Does fixed-deadline auction of a *public-value* good attract more sellers of high quality objects than one in which bidding is extended until no one wants to bid anymore? (Because the latter deters experts from bidding?)

Naive bidding and best response to naive bidding (with Oct 12, 2000 enhancement to bid history data)
Some final, methodological thoughts:

Robustness and fragility:

The theoretical result that rational collusion is possible (on eBay) is robust, but the theorem that it is impossible (on Amazon) is fragile.

That is, the possibility result shows that late and multiple bidding isn’t inconsistent with rationality. This doesn’t depend on equilibrium—it makes just as much
sense e.g. in response to a naïve English auction bidder.
The impossibility result is a strong theorem, and as such it is in part a fixed point involving the exact statement of the theorem (undominated strategies, sequential rationality) and the exact statement of the model.

**Eclectic Methodology**
We use theory, field data, surveys, laboratory experiments…

**Multiple identical items, one per winner**

4. Uniform pricing > discriminatory pricing

5. In sequential auctions, upward-drifting prices

**Multiple items, multiple items per winner**
(“combinatorial” auctions)

6. Vickrey (sealed-bid) procedure is strategically transparent

**The Vickrey (-Clarke-Groves) Combinatorial Auction Procedure**
Two items, A and B, are up for sale.

Bidder 1 wants only item A, and values the two items and the package at

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>AB</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>150</td>
<td>0</td>
<td>150</td>
</tr>
</tbody>
</table>

Bidder 2 wants only item B, and values the two items and the package at

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>AB</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
<td>150</td>
<td>150</td>
</tr>
</tbody>
</table>

Bidder 3 wants only the package, and values the two items and the package at

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>AB</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
<td>0</td>
<td>200</td>
</tr>
</tbody>
</table>

Assume that each submits bids on the individual items and the package equal to their valuations.

Giving A to bidder 1 and B to bidder 2 maximizes the total bid at 300. If either bidder had not been present, the maximizing allocation would give AB to bidder 3 with a total bid of 200.

**The VCG allocation rule**: Each bidder receives the package assigned to him in the maximizing-total-bid partition.
Bidder 1 gets A  
Bidder 2 gets B

The VCG pricing rule: Each winning bidder pays the amount he bid for the package received, and is rebated the increment to the maximizing total bid caused by his participation. (Note that, when only one item is up for sale, this is just the second-price auction procedure; when many identical items are being sold, it is the uniform-price procedure.)

Therefore:

Bidder 1 pays \(150-(300-200) = 50\)
Bidder 2 pays \(150-(300-200) = 50\)

The remarkable VCG property: Each bidder, given that the other bidders bid their valuations, can do no better than to bid his own valuations. (In economic parlance, the procedure is a “demand-revealing mechanism.”)

**The “Political” Problem with VCG**

Bidder 1 values the two items and the package at

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>AB</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bid</td>
<td>160</td>
<td>0</td>
<td>160</td>
</tr>
</tbody>
</table>
Bidders 2 and 3 are as before.

Again, each submits bids on the individual items and the package equal to their valuations.

Bidder 1 gets A, pays $160-(310-200) = 50$
Bidder 2 gets B, pays $150-(310-200) = 40$ (!!!)