Recovering Credit Portfolio Loss Rates from CDO Tranches: Solution of an inverse problem by intensity control

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The calibration of (top-down) pricing models for portfolio credit derivatives such as CDOs involves the construction of a risk-neutral jump intensity for the loss process which is compatible with a set of observations of market spreads for CDO tranches. We propose an efficient and stable algorithm to solve this inverse problem by transforming it into a stochastic control probem. Given a set of observations of market spreads for CDO tranches, we construct a risk-neutral default intensity process for the portfolio underlying the CDO which matches these observations, by looking for the risk neutral loss process verifying the calibration constraints 'closest'
--in the sense of relative entropy-- to a prior loss process. We formalize the problem in terms of minimization of relative entropy with respect to the prior under calibration constraints and use convex duality techniques to solve the problem. The dual problem is shown to be an intensity control problem, characterized in terms of a nonlinear Hamilton Jacobi system of differential equations. We show that the Hamilton Jacobi system can be solved explicitly thus yielding an efficient method for obtaining the calibrated intensity. Our method allows to construct an implied intensity process which leads to CDO tranche spreads consistent with the observations. We illustrate our method ITRAXX index data: our results reveal strong evidence for the dependence of loss transitions rates on the past number of defaults, thus offering quantitative evidence for ``contagion effects'' in the risk-neutral loss process.